PATTERNS OF WAR TERMINATION:
A STATISTICAL APPROACH

THESIS

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Abstract

This research uses an advanced statistical technique to expand upon the current understanding of war termination. Specifically, this thesis addressed questions concerning the most relevant factors toward predicting both the outcomes of interstate wars and the winners of intrastate and extra-systemic wars, within the limitations of the available data. Open-source war data from the Correlates of War Project was analyzed using both binary and multinomial logistic regression techniques. While the Correlates of War Project did not necessarily focus its data collection efforts on those variables historically associated with war termination, it did provide a sufficient number of variables with which to demonstrate the applicability of logistic regression techniques to war termination analyses. As a consequence, every significant logistic regression model contains a single relevant variable. For both intrastate and extra-systemic wars, the duration of the conflict was found to be most relevant to predicting the winner. In contrast, the proportion of total casualties borne by a nation in an interstate war was most relevant to predicting the manner in which an interstate war ends. Conclusions drawn from this research and suggestions for future statistical applications to war termination studies were also discussed.

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I. Introduction

Background

What must be done to convince an enemy to give up armed resistance? Most of the research on wars has been devoted to the prevention of war. Much less focus has been placed on studying the factors involved in terminating a war after it ensues (Pillar, 1983:3).

Problem Statement

Permeating throughout war termination literature is the lesson that deciding how a war shall end is just as important as deciding how a war shall be fought (Ikle, 1991:1). Additionally, ending a war such that the desired state of peace is achieved is equally paramount. Knowledge must be gained concerning the appropriate amount of military force required, not only to affect the cessation of hostilities, but also to contribute positively to the planned peace (Ikle, 1991:x). Under the assumption that war is a complex and unstable phenomenon, it is appropriate to examine war termination through a probabilistic lens. What factors are relevant to ceasing armed hostilities? To what degree are such factors significant? Can these factors be controlled or manipulated? Given specific values for such relevant factors, what is the likelihood of achieving one

outcome versus another? Logistic regression analyses on historical war data can address these questions and provide objective insights into existing social science theories.

Numerous theories on war termination exist, and they have been used in political and social science circles to explain the outcomes of past wars. However, beyond elementary statistical measures, such as the proportion of wars since 1815 ending by a negotiated settlement, there appears to be a lack of rigorous applications of advanced statistical methods to describe how wars end. As a consequence, few of the social science theories on war termination can be consistently applied, given similar wartime conditions in multiple cases. Authors of these war termination studies suggest many methods to devise a successful termination strategy, but few numerical methods have been employed to either support or contradict their arguments.

Research Objectives

This thesis sought to identify the key factor or factors that contribute to the termination of an armed conflict using readily available open-source data. The overarching goal was to demonstrate the applicability of logistic regression techniques to war termination analyses. Once the key variables were identified, the next phenomenon to be addressed was how the contributory factors influence trends in both how wars end and who wins wars. The three types of wars were analyzed separately to identify different war termination patterns between war types. This study also sought to identify developing trends between 19^{th} and 20^{th} Century warfare because the open-source data used in this study spanned these two centuries. One such pattern is the change in likelihood, from Napoleonic to modern warfare, that a particular combatant wins a war.

The change in the likelihood of a particular outcome between centuries was also of interest. Any wars found to have significant effects on estimating the models were identified for future research.

Limitations

The data sets used for this research were obtained from the Correlates of War Project (COWP). The COWP is based in Urbana, IL, and consists of scholars, mostly political scientists, devoted to increasing the scientific exploration and knowledge of war. The group was founded in 1963 by political scientist J. David Singer, who was soon joined by historian Melvin Small. The data sets compiled by the COWP consist primarily of variables determined by the group to be relevant to the onset of war, such as international trade, nonaggression pacts, defense alliances, geographic contiguity, national materiel production, and diplomatic representation.

The small number of variables for which the COWP collected data limited the discovery of a comprehensive list of statistically significant war termination factors. This limitation also restricted the size and implications of the resulting logistic regression models. There are more variables discussed in the existing war termination literature than were variables within the COWP data. Consequently, some of the insights gained from the social science realm remain open to further investigation.

The data sets available from the Correlates of War Project, which also included data concerning diplomatic ties, trade agreements, and alliances in addition to war data, were compiled by different persons. Therefore, it was difficult to pinpoint similarities

between data sets. The ability to add and delete variables between data sets such that the models are better specified also requires additional investigations.

Numerous missing entries existed within each of the data sets. While valid statistical techniques can be used to fill in missing data, the resulting analyses would be more useful in real-world applications if the data were complete. The sample sizes for each of the three data sets analyzed, on the other hand, were sufficiently large such that the observations containing missing data could be deleted with little effect on the model parameter estimates.

Research Focus

This research focused on the analyses of data concerning three types of wars: interstate, intrastate, and extra-state or extra-systemic. The data were further distinguished by century. That is, the data for each war type were further divided into 19^{th} and 20^{th} Century data. Interstate wars are those whose participants are internationally recognized nations. Intrastate wars are defined as armed conflicts involving belligerents confined within a nation's geographic borders, including civil wars. Extra-state or extra-systemic wars are those involving state and non-state actors, but fighting occurs outside the nation's borders. The terms extra-state and extra-systemic both define the same type of war, so they are used interchangeably throughout this thesis.

A review of existing social science literature on war termination was conducted. The level of attention previously devoted to the subject of war termination was addressed. The literature review also discussed the subjective methods used in prior studies to

classify the types of war termination. These prior classifications provided a basis from which to construct the war termination categories used in this study.

Two sources of logistic regression theory were reviewed. The work of Hosmer and Lemeshow explained virtually all of the techniques and methods used in logistic regression. The contribution by Montgomery, Peck, and Vining to this study was a thorough description of the least squares method used to estimate the logistic regression model parameters.

Subsets of variables from the original COWP data were selected. These selections were made based primarily on relevant factors discussed in the social science literature on war termination. Additionally, the sets of selected variables were further limited by variable availability in the COWP data. That is, several factors deemed important by social scientists were not available in the COWP data. The variable restriction, however, did not adversely affect the overall intent of this study, which was to demonstrate the applicability of logistic regression techniques to war termination problems. A sufficient number of variables were provided by the COWP such that the effectiveness and potential of logistic regression applications to war termination could be shown.

Stepwise selection is a robust procedure that was used to determine an initial set of statistically significant variables for each fitted model. Stepwise selection was conducted on the variables for the 19th Century, 20th Century, and aggregated data for each type of war. The results from the stepwise procedure were used to estimate initial logistic regression models. The initial models were each assessed for goodness-of-fit and

individual covariate significance. The significance tests confirmed either the overall adequacy of an initial model or the need to fit a reduced model.

The statistical software program used in this study was MINITAB. Several software packages have been programmed to fit and analyze logistic regression models, but MINITAB was chosen for two reasons. One, MINITAB was readily available and accessible. Secondly, MINITAB had been programmed to support binary logistic regression, multinomial logistic regression, and virtually all of the significance tests, goodness-of-fit tests, diagnostic measures, and diagnostic plots necessary for this investigation.

Each of the final models was assessed for overall adequacy using three statistically equivalent goodness-of-fit tests. Individual covariate significance was also determined through tests on their coefficients. The implications of each model were also interpreted. Three types of residual plots were examined for influential observations. Once identified, the influence points were analyzed for their net effects on model coefficient estimations. When necessary, new models were fit with the influential data points deleted.

A general assessment of the findings of this study was given. War termination implications across two centuries of warfare and across three types of wars, given the open-source data used, were stated. Opportunities for future statistical studies on war termination were considered. In addition, proposals for additional applications of logistic regression methods to war termination were discussed.

II. Literature Review

General

Few will deny that all wars do not end in the same manner, yet not enough attention is paid to the elements contributing to the conclusion of wars. Fred Ikle addresses the one-way street between how wars begin and how they end, and he insists that the process of termination has the longest lasting effect on the ensuing peace than any other element of war (Ikle, 1991:vii). One need look no further than to German actions during World War I and to French actions after World War I to accept Ikle's assessment as an axiom of war. Germany launched its unrestricted submarine warfare campaign in 1916 with the intent to inflict massive panic upon the British population and end the war on German terms, but the campaign instead served the unintended consequence of drawing the United States into the war, which hastened Germany's defeat (Ikle, 1991:xi). Germany's perceived military excesses during World War I led to French insistence that the Versailles treaty punish Germany economically through massive war reparations and humiliate Germany diplomatically by forcing her to accept the aggressor label. The eventual rise of Adolf Hitler and Nazi Germany can be traced back, at least in part, to French contributions to the Treaty of Versailles.

Classifying the manners in which wars end is important to a probabilistic analysis of war termination. Paul Pillar conducts such a classification in his analyses. However, he postulates that most future wars will end through negotiated agreements, so his classification of the types of war termination is influenced by this assertion. It must first be determined whether combat ends at the same time as the war (Pillar, 1983:11). For

example, Serbia and Turkey signed a peace treaty in March 1877, which technically ended the First Balkan War, but some Serbian forces continued to fight the Turks through the beginning of the Russo-Turkish War in April 1877 (Pillar, 1983:22). Pillar classifies this type of war termination as absorption. That is, the ending of a small war is marked by one or more of its belligerents becoming involved in a larger war. If combat does indeed end simultaneously with the war, then it should be determined whether the fighting ended because of a mutual agreement by all belligerents or because one side applied sufficient military force to the opposition such that its enemy could no longer continue. If the latter is the case, then Pillar denotes this type of war termination as extermination or expulsion. When all sides mutually decide to end the war, then Pillar notes either the existence or absence of a written agreement. Pillar defines withdrawal as a war which terminates without a written agreement (Pillar, 1983:14).

For explicit agreements, Pillar distinguishes between those negotiated by the belligerents themselves and those negotiated by third parties. Pillar further assumes that international organizations have almost always played the role of the third party in written negotiations. As such, he uses the term international organization to denote the category for wars in which a third party aids in written agreements (Pillar, 1983:15).

When formal settlements are handled by the belligerents themselves, Pillar discerns whether or not a settlement is imposed by one side upon the other. If this is the case, then capitulation has occurred. If the settlement is indeed mutually negotiated, then Pillar differentiates between agreements negotiated before an armistice and those negotiated after an armistice (Pillar, 1983:15). These distinctions add support to the

construction of a polychotomous, or multi-category, dependent variable on the outcomes of wars.

With the response variable defined, the focus of investigation must necessarily shift towards the common factors that contribute to stopping a given war. Additionally, attention should be given to the manner in which a war ends, not just why it ends. For example, the proportion of total casualties taken by one belligerent may prove to be more significant if the war ends through capitulation than if it ends through a negotiated settlement. Because every war is different, only a few termination variables are present in all wars.

Ikle points out the obvious economic and social costs of casualties and military expenditures (Ikle, 1991:1). Even with the ongoing Operation Iraqi Freedom (OIF), the most commonly cited measures are the numbers of US dead and wounded, Iraqi civilian deaths, and the billions of dollars per month spent on the conflict. Most other factors mentioned in the literature are qualitative in nature. As a consequence, limited data is available for these factors, and their relevance is largely based on hindsight, conjecture, and inference.

There does exist at least one case where these subjective variables are applied to social science war termination theories using what could be considered survey data as supporting evidence. Joseph Engelbrecht, in his analyses of four war termination theories, uses transcripts from interviews with Japanese officers captured during World War II to support his conclusions (Engelbrecht, 1992:82-87). His conclusions, however, seek to explain why wars end rather than to relate the relevant factors to specific types of war endings. His case-study approach only addresses one type of war termination:

surrender or capitulation. In two of the three cases, the Japanese surrender in 1945 and the Afrikaner surrender to the British in South Africa in 1902, a formal settlement to the conflict was reached.

Two interesting political science theories on war termination are considered by Engelbrecht and tested against three cases. One theory is based on a winners and losers approach. The other focuses on cost/benefit analyses. The three test cases he used were the Japanese decision to surrender in August 1945, the Afrikaner decision to surrender to the British in South Africa in 1902, and the British decision to continue fighting the Nazis following the fall of France in 1940. He applied each theory to each case, analyzed the particulars of each case, and determined which theory best fit the decisions made in each case (Engelbrecht, 1992:61-63).

The Winners and Losers model identifies two outcomes of war and emphasizes that one side is the clear victor, and the other side is the vanquished. This model stresses the defeat of enemy military forces as the key to convincing the enemy to either seek a peaceful settlement or surrender. This theory is commonly applied when one can identify a specific battle or campaign that marks a turning point in the war (Engelbrecht, 1992:63-64).

For example, the the Battle of Midway in 1943 is identified as the battle that turned the tide of World War II against Imperial Japan. Interrogations of Imperial Japanese military officers at the end of World War II confirmed that the American victory at Midway signaled the eventual defeat of Japan (Engelbrecht, 1992:82-87). In the Afrikaner case, the fall of Pretoria in 1900 turned the tide of the Anglo-Boer War against the Boer forces (Engelbrecht, 1992:155-157).

The German blitzkrieg through the Ardennes, the defeat of the British Expeditionary Force (BEF) in Belgium, and the fall of France were devastating defeats to the United Kingdom in 1940, yet the British refused to negotiate or surrender. However, the defeated nation must capitulate soon after such turning points in order for the Winners and Losers theory to be valid (Engelbrecht, 1992:215). In all the cases described above, the defeated nation did not immediately surrender, despite heavy battlefield losses. The Afrikaners did not surrender to the British until 1902. The Japanese surrender did not come until 1945, yet the interrogated Japanese officers deemed the surrender inevitable, even without the atomic bomb attacks on Hiroshima and Nagasaki. On the other hand, the British never surrendered or talked of peace with Nazi Germany. Why? Why did surrender eventually occur in all the other cases, except the British? The same conditions of a humiliating military defeat existed in all the cases, yet surrender did not always occur.

The Cost Benefit model focuses on comparing the costs of prosecuting a war with the achievement of the war's objectives. For this theory to be applicable, the losing nation is expected to first weigh the costs of war. That is, it must consider the raw numbers of human, war weapon, logistic, and economic losses. Then, the losing nation must determine whether or not its war aims can still be reasonably met. If its war objectives cannot reasonably be met, then the Cost Benefit model implies that capitulation must occur (Engelbrecht, 1992:30-32). In all three cases analyzed by Engelbrecht, no evidence suggested the use of any rational cost benefit analyses to decide the question of war termination, at least while the war was ongoing. That is not to say

that costs were not discussed, but such discussions did not directly produce a decision to surrender, or in the British case, to continue fighting (Engelbrecht, 1992:32-33).

James Walker begins his Naval War College study by addressing the question of why war termination plans should be considered. He notes that the majority of wars since 1800 have ended with negotiated peace agreements. This fact moves the purpose of military force away from the wholesale destruction of enemy forces on the battlefield and toward the application of sufficient force to achieve diplomatic and political goals. He points to the numerous Arab-Israeli wars to support the idea of this paradigm shift. The undefeated military record of Israel, most notably in its War of Independence in 1948, the Six Day War in 1967, and the Yom Kippur War in 1973, has achieved neither a lasting peace nor a resolution of the political, social, and religious issues between Israel and her Arab neighbors. Dynamic political, diplomatic, social, and cultural issues lend even more importance to war termination planning (Walker, 1996:1-2).

Walker notes that war termination is mentioned in the joint military doctrine of the United States, but the attention it is given is brief and the language vague. He describes a state of tunnel vision resulting from America's status as the lone superpower. That is, military commanders falsely assume that the mere overwhelming application of America's superior weapons and firepower will automatically produce the desired peace (Walker, 1996:2-4). This assessment essentially echoes a similar statement made by Ikle, where Ikle asserted that military power should be applied only to the extent that it will contribute positively to the desired peace, and such applications should be explicitly defined in military strategies for war. Ikle maintains that the indiscriminant destruction

of enemy forces and civilians is most detrimental to the desired atmosphere of peace (Ikle, 1991: ix-xi).

The products of termination agreements must be considered. Will written documents be drafted and signed by all parties? If so, will it be a formal treaty? If not a treaty, will it be an armistice or limited cease-fire? Walker highlights these details for two reasons. One, the Gulf War negotiations yielded no written agreements, only audio recordings. Two, Walker emphasizes the international legitimacy behind written agreements. Although only treaties are legally binding, written agreements, in general, still provide a certain degree of political and diplomatic leverage in the event that one side eventually breaks the deal (Walker, 1996:12-13). Unlike Pillar, Walker treats armistices and cease-fires as actual termination agreements rather than conditions upon which formal war settlements hinge.

Emphasizing the importance of war termination in both doctrine and training is the method Walker offers with respect to how to plan for war termination. Beyond that, he only stresses drafting war termination plans early in the strategic planning cycle. As with other operational plans, war termination plans should be updated according to the progression of affairs in the war. Alternatives within the termination plans should be analyzed, and contingencies should also be considered (Walker, 1996:13-14). Rather than provide guidance on war termination methods, Walker focuses on the lack of attention given to and the necessity for early planning of war termination (Walker, 1996:16).

Correlates of War Project (COWP)

The COWP is an organization that provides open-source data on wars and factors which account for wars. The COWP has compiled thirteen data sets. These sets contain variables concerning state system membership, interstate wars, intrastate wars, extra-systemic wars, militarized interstate disputes, national materiel capabilities, formal alliances, territorial changes, geographic contiguity, colonial dependency, intergovernmental organizations (IGOs), diplomatic representation, and bilateral trade. In the context of a war termination study, the interstate, extra-state, and intrastate war sets are of primary interest. The interstate set contains data concerning the nations participating in 79 interstate wars from 1823 to 1991. The intrastate set contains data concerning the state belligerents in 213 intrastate wars from 1816 to 1997. The extra-state set contains data concerning the state actors in 108 extra-systemic wars from 1817 to 1983. Appendix A shows the variables included in each of the three war data sets and their definitions as assigned by the COWP.

Statistical Application

Suppose the response variable in a statistical study on war termination is the winner of a war. Either a particular combatant wins, or his opponent does. He succeeds in defeating his opponent or his enemy defeats him. Since this response has only two possible outcomes, and its category definitions are arbitrary, the winner of a war can be defined as a Bernoulli random variable (Montgomery, Peck, and Vining, 2001:443-444). That is, each category for the winner has a probability attached to it. As a contemporary example, let Y_j denote the winner of the j^{th} extra-systemic war, which involved the

United States and the terrorist group Hamas. Let j denote the j^{th} extra-state war from a sample of n extra-state wars, where $j = 1, 2, \ldots, n$. If $Y_j = 0$, then Hamas is the winner. If $Y_j = 1$, then the United States is the winner. Since Y_j is a Bernoulli random variable, the probability that $Y_j = 0$ and the probability that $Y_j = 1$ are the quantities under investigation (Montgomery, Peck, and Vining, 2001:444). The goal now is to determine a mathematical relationship between who wins an extra-systemic war and appropriate contributory or predictor variables.

Alternatively, suppose the response variable in a statistical study on war termination is the manner in which a war terminates. More than two types of war termination have been defined to exist, so the response is polychotomous or multi-category. The probabilities for the different types of war termination are still of interest, but each war termination probability is compared to a reference or baseline war termination probability (Hosmer and Lemeshow, 2000:260-261). That is, the type of war termination that is most prevalent is selected to be the reference category, and the remaining categories are compared to it. Mathematical relationships between each comparison and several predictor variables can now be established. In this case, the objective can be to determine how likely one type of war termination is to occur over the baseline war termination method (Hosmer and Lemeshow, 2000:265).

Once the response is identified and its structure defined, a set of candidate predictor variables is compiled. Advanced statistical techniques can be applied to these candidate variables to determine the strengths of their relationships to the response. The results from such techniques can justify the retention or elimination of some of the candidate variables.

Logistic Regression

Because this thesis focuses on analyses performed on existing data, a regression technique is an effective way of describing the relationship between how a war ends, or who wins a war, and the factors contributing to such outcomes. The outcome of a war is not a continuous variable, so classical linear regression is not a valid approach. Instead, this thesis seeks to assess the likelihoods of different outcomes of war, and such likelihoods can be derived from conditional probabilities. Logistic regression is the preferred method for this approach, primarily because the outcome variables are discrete categorical variables, either binomial or multinomial (Hosmer and Lemeshow, 2000:1). Some texts use the synonymous terms binary or dichotomous when referring to a logistic regression model with a two-category response. They also may use the terms polychotomous or polytomous when referring to a logistic regression model with a response containing three or more categories (Hosmer and Lemeshow, 2000:260).

The nature of the response variable determines the type of parametric model to be used. It also determines the assumptions that can be made. In linear regression, the response is continuous, and the distribution of the response is assumed to be normal. The outcome of a war, however, is not a continuous random variable as defined in this study. Similarly, the winner of a war is not a continuous random variable. Thus, the normality assumption no longer holds for the responses in this study. These responses must be described by a different probability distribution (Hosmer and Lemeshow, 2000:1).

As with linear regression, model parsimony is also desired with logistic regression. That is, fitting the model with the smallest number of contributory variables,

or covariates, that best describes the relationship between an outcome, or response, and a set of covariates, or predictors (Hosmer and Lemeshow, 2000:1). The model can contain either continuous variables, categorical variables, or both.

Binary Logistic Regression.

The theory behind binary logistic regression is commonly explained using a univariate model, where only one covariate is present. The techniques are readily adapted to multivariate cases. The focal quantity for binary logistic regression is the conditional probability of the mean of the response, given a certain value of the covariate. That is, $P(Y=i \mid x=j)$. Several cumulative distributions have been proposed and used to fit models for this conditional probability, but the logistic distribution is used for logistic regression because of its ease of interpretation. The binary logistic regression model is of the form

$$\pi(x)=\frac{e^{\beta_0+\beta_1 x}}{1+e^{\beta_0+\beta_1 x}}, \tag{2.1}$$

where $\pi(x)=P(Y \mid x)$ represents the conditional probability of the response Y given the covariate x (Hosmer and Lemeshow, 2000:6). For the multivariate case, let $\mathbf{x}^T=[1, x_1, x_2, \ldots, x_k]$ and $\boldsymbol{\beta}^T=[\beta_0, \beta_1, \beta_2, \ldots, \beta_k]$. Then, the multivariate logistic regression model becomes

$$\pi(\mathbf{x})=\frac{e^{\mathbf{x}^T\boldsymbol{\beta}}}{1+e^{\mathbf{x}^T\boldsymbol{\beta}}}. \tag{2.2}$$

The method of maximum likelihood is used to estimate the parameters of the model, but the model must be transformed and made linear in its parameters β_0 and β_1. The transformation used is called the *logit*. The logit is defined in terms of $\pi(x)$.

$$g(x) = \ln\left(\frac{\pi(x)}{1-\pi(x)}\right) = \beta_0 + \beta_1 x \tag{2.3}$$

For multiple covariates, the logit becomes

$$g(\mathbf{x}) = \ln\left(\frac{\pi(\mathbf{x})}{1-\pi(\mathbf{x})}\right) = \mathbf{x}^T\boldsymbol{\beta} = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \cdots + \beta_k x_k \tag{2.4}$$

It should be noted that the quantity $\pi(x)/(1-\pi(x))$ is called the *odds*, that is, the ratio of the probability of success to the probability of failure. Therefore, the logit is also called the *log-odds* (Montgomery, Peck, and Vining, 2001:445-446).

An observation of a dichotomous response given x is expressed as $y = \pi(x) + \varepsilon$, but the assumption of normality in the distribution of the error term ε does not apply in this case, as it does in linear regression (Hosmer and Lemeshow, 2000:6). Instead, the errors follow the binomial distribution, with a mean or expected value of zero and a variance equal to the product of the probability that $y = 1$ and the probability that $y = 0$. That is,

$$\varepsilon = 1 - \pi(x) \text{ with probability } \pi(x), \text{ for } y = 1, \tag{2.5}$$

$$\varepsilon = -\pi(x) \text{ with probability } 1 - \pi(x), \text{ for } y = 0, \tag{2.6}$$

$$E(\varepsilon) = 0, \text{ and} \tag{2.7}$$

$$Var(\varepsilon) = \pi(x)\left[1 - \pi(x)\right]. \tag{2.8}$$

Constructing the likelihood function is the first step towards estimating the logistic regression model parameters. Let (x_j, y_j) denote one observation out a set of n independent observations, where y_j is the j^{th} binary response, x_j is the value of the covariate for the j^{th} observation, and $j = 1, 2, \ldots, n$ (Hosmer and Lemeshow, 2000:7). The contribution of (x_j, y_j) to the likelihood function is expressed as an independent Bernoulli trial, or

$$\pi\left(x_j\right)^{y_j}\left[1-\pi\left(x_j\right)\right]^{1-y_j} = \pi_j^{\;y_j}\left(1-\pi_j\right)^{1-y_j} \tag{2.9}$$

Since there are n independent Bernoulli trials, and each trial contributes to the likelihood function, then the likelihood function becomes the product of independent trials, or

$$l(\beta_0, \beta_1) = \prod_{j=1}^{n} \pi_j^{\;y_j}\left(1-\pi_j\right)^{1-y_j} \tag{2.10}$$

In order to find the values of β_0 and β_1 that maximize equation (2.10), the natural logarithm of equation (2.10), the *log-likelihood function*, is computed because it is easier to manipulate (Hosmer and Lemeshow, 2000:8). Differentiating the log-likelihood function $L(\beta_0, \beta_1)$ with respect to β_0 and β_1, and setting each resulting partial differential equation to zero, yields the *likelihood equations*.

$$L(\beta_0, \beta_1) = \sum_{j=1}^{n}\left[y_j \ln\left(\pi_j\right) + \left(1-y_j\right)\ln\left(1-\pi_j\right)\right] \tag{2.11}$$

$$\sum_{j=1}^{n}\left(y_j - \pi\left(x_j\right)\right) = 0 \tag{2.12}$$

$$\sum_{j=1}^{n} x_j\left(y_j - \pi\left(x_j\right)\right) = 0 \tag{2.13}$$

Using vector notation, the form of the log-likelihood function for multivariate cases is

$$L(\boldsymbol{\beta}) = \sum_{j=1}^{n} y_j \mathbf{x}_j^T \boldsymbol{\beta} - \sum_{j=1}^{n} \ln\left[1 + \exp\left(\mathbf{x}_j^T \boldsymbol{\beta}\right)\right] \tag{2.14}$$

(Montgomery, Peck, and Vining, 2001:448). Because the likelihood equations are nonlinear in their parameters, a closed-form solution is not possible. An iterative search method called iteratively reweighted least-squares (IRLS) is implemented to obtain solutions (Hosmer and Lemeshow, 2000:9).

Most modern statistical software packages that fit logistic regression models have this iterative search method programmed into them. IRLS employs the Newton-Rhapson algorithm as a robust method to approximate solutions to the likelihood equations. Hosmer and Lemeshow do not describe the details of IRLS, but the interested reader should refer to Montgomery, Peck, and Vining for a complete explanation of IRLS (Montgomery, Peck, and Vining, 2001:610-613).

Let $\hat{\boldsymbol{\beta}}$ be the final IRLS estimate. Then, the logit becomes $\hat{g}(\mathbf{x}_j) = \mathbf{x}_j^T \hat{\boldsymbol{\beta}}$, and the fitted logistic regression model becomes

$$\hat{\pi}_j = \frac{\exp\left(\mathbf{x}_j^T \hat{\boldsymbol{\beta}}\right)}{1 + \exp\left(\mathbf{x}_j^T \hat{\boldsymbol{\beta}}\right)} \tag{2.15}$$

(Montgomery, Peck, and Vining, 2001:449).

Parameter Interpretation.

For the binary model, the fitted value of its logit at a particular value of its single covariate is $\hat{g}(x_j) = \hat{\beta}_0 + \hat{\beta}_1 x_j$. Let the value of the logit at $x_j + 1$

be $\hat{g}\left(x_j+1\right)=\hat{\beta}_0+\hat{\beta}_1\left(x_j+1\right)=\hat{\beta}_0+\hat{\beta}_1+\hat{\beta}_1 x_j$. Therefore, the difference between the two fitted logit values is

$$\hat{g}\left(x_j+1\right)-\hat{g}\left(x_j\right)=\hat{\beta}_0+\hat{\beta}_1+\hat{\beta}_1 x_j-\hat{\beta}_0-\hat{\beta}_1 x_j=\hat{\beta}_1$$

or

$$g\left(x_j+1\right)-g\left(x_j\right)=\ln\left[\frac{\pi\left(x_j+1\right)}{1-\pi\left(x_j+1\right)}\right]-\ln\left[\frac{\pi\left(x_j\right)}{1-\pi\left(x_j\right)}\right]=\ln\left(\frac{odds_{j+1}}{odds_j}\right)$$

If the antilogarithm of the above quantity is taken, then the result is called the *odds ratio*,

$$\hat{O}_R=\frac{odds_{j+1}}{odds_j}=e^{\hat{\beta}_1} \tag{2.16}$$

which is the estimated change in π per one-unit change in the covariate x. For multivariate models, $\hat{O}_R$ is the estimated change in π per one-unit change in the j^{th} covariate, given that the values for the remaining $k-1$ covariates are constant (Montgomery, Peck, and Vining, 2001:452).

Odds ratios, rather than the parameter estimates, are used to describe the results of a fitted binary logistic regression model. For example, suppose that a binary logistic regression model on the winner of an intrastate war contains the length of the conflict as the predictor variable, and suppose Y denotes the binomial random variable for the winner. Let $Y=0$ denote that the state actor wins the intrastate war, and let $Y=1$ denote that the non-state actor, rebel faction, or insurgency wins the war. In addition, suppose that 2.5 is found to be the odds ratio for this model when the duration of the war is 1440 days. It can then be said that the non-state belligerent is two and a half times more likely to win an intrastate war than is the state, given that the war lasts 1440 days.

Goodness-of-Fit Testing.

Measuring the difference between observed and fitted values, or residuals, to assess a model's goodness-of-fit can be performed by manipulating *likelihood ratios*. That is, the IRLS estimates for the parameters in equation (2.3) are substituted into the log-likelihood function (2.11), which maximizes the value of the log-likelihood function. By noting that a saturated model is one whose sample size is equal to the number of parameters it contains, or $n = k + 1$, the difference between the log-likelihood of this saturated model and the log-likelihood of the fitted model is examined to determine the fitted model's adequacy.

The *deviance D* of the fitted model approximately possesses a chi-square distribution with $n - (k+1)$ degrees of freedom. The test statistic is given by

$$D = 2\ln\left(\frac{l(saturated)}{l(\hat{\boldsymbol{\beta}})}\right) = 2\left(L\left(saturated\right) - L\left(\hat{\boldsymbol{\beta}}\right)\right) \tag{2.17}$$

Multiplying the natural logarithm of the likelihood ratio by 2 allows the deviance to approximate a chi-square distribution (Hosmer and Lemeshow, 2000:13). If $D \le \chi^2_{\alpha,n-k-1}$, then the fitted model is appropriate; $D > \chi^2_{\alpha,n-k-1}$ implies that the fitted model is incorrectly specified (Montgomery, Peck, and Vining, 2001:453). The quantity α is the specified level of significance; 0.05 is the α level used for this research.

The second commonly conducted test is the Pearson chi-square statistic. Let J be the number of distinct values of the covariate observed in the data set, and let m_j be the frequency of the j^{th} distinct covariate value, where $j = 1, 2, \ldots, J$. For the purpose of

computing Pearson residuals, let y_j be the frequency of the j^{th} distinct covariate value for which $y = 1$. It follows that the sum of the m_j fitted values is

$$m_j \hat{\pi}_j = m_j \frac{\exp\left(\hat{g}\left(x_j\right)\right)}{1 + \exp\left(\hat{g}\left(x_j\right)\right)} \tag{2.18}$$

Thus, the Pearson residual for the j^{th} distinct covariate value is given by

$$r\left(y_j, \hat{\pi}_j\right) = r_j = \frac{y_j - m_j \hat{\pi}_j}{\sqrt{m_j \hat{\pi}_j \left(1 - \hat{\pi}_j\right)}} \tag{2.19}$$

The Pearson chi-square statistic, X^2, is the sum of the squares of the Pearson residuals.

$$X^2 = \sum_{j=1}^{J} r\left(y_j, \hat{\pi}_j\right)^2 \tag{2.20}$$

As implied by its name, the Pearson chi-square statistic follows a χ^2 distribution with $J - \left(k + 1\right)$ degrees of freedom. The fitted model is said to be correctly specified if $X^2 \leq \chi^2_{\alpha, J-k-1}$ (Hosmer and Lemeshow, 2000:145-146).

To conduct the Hosmer-Lemeshow test, the observations are grouped using the following method. Ten groups are created such that each group contains approximately $n_i^{'} = n/10$ fitted values, where $i = 1, 2, \ldots, 10$. The groups are tabulated in order of increasing fitted value. That is, there are $n_1^{'}$ subjects with the smallest fitted values in group 1, while there are $n_{10}^{'}$ subjects with the largest fitted values in group 10. The groups serve as the columns of a 2×10 table, where the rows are denoted by the two possible values of the dichotomous response. For the $y = 1$ row, the expected frequencies for each group are computed as follows:

$$\sum_{j=1}^{n_i'} \hat{\pi}_{ij}\ , \qquad \text{for}\ i = 1, 2, \ldots, 10\ . \tag{2.21}$$

Conversely, the expected frequencies for each group in the $y = 0$ row are given by

$$\sum_{j=1}^{n_i'} \left(1 - \hat{\pi}_{ij}\right) \tag{2.22}$$

It is necessary to develop the elements of the Hosmer-Lemeshow statistic before stating its formula. Let c_i be the number of distinct covariate values in the i^{th} group, and

$$o_i = \sum_{j=1}^{c_i} y_j$$

is the sum of the number of responses over all distinct covariate values in the i^{th} group. The average fitted value is

$$\overline{\pi}_i = \sum_{j=1}^{c_i} \frac{m_j \hat{\pi}_j}{n_i'}\ \ ,$$

and the Hosmer-Lemeshow statistic, $\hat{C}$, is given by

$$\hat{C} = \sum_{i=1}^{10} \frac{\left(o_i - n_i' \overline{\pi}_i\right)^2}{n_i' \overline{\pi}_i \left(1 - \overline{\pi}_i\right)}\ . \tag{2.23}$$

The use of 10 groups is not universal. If the number of distinct covariate values is small or very large, then adjusting the number of groups may be necessary. According to Hosmer and Lemeshow, the use of 10 groups provides an adequate approximation to the chi-square distribution in most applications (Hosmer and Lemeshow, 2000:148-149). In this case, the Hosmer-Lemeshow statistic is distributed chi-square with $10 - 2 = 8$ degrees of freedom.

Diagnostic Measures.

As with linear regression, leverage values for logistic regression are also derived from a *hat matrix*, $\mathbf{H}$. Let $\mathbf{V}$ be a $J \times J$ diagonal matrix whose j^{th} diagonal element is given by

$$v_j = m_j \hat{\pi}_j \left(1 - \hat{\pi}_j\right).$$

Let the design matrix, $\mathbf{X}$, be the $J \times (k+1)$ matrix containing all distinct covariate values. The hat matrix is defined by

$$\mathbf{H} = \mathbf{V}^{1/2}\mathbf{X}\left(\mathbf{X}^T\mathbf{V}\mathbf{X}\right)^{-1}\mathbf{X}^T\mathbf{V}^{1/2} \qquad (2.24)$$

It follows that the hat matrix in equation (2.24) is also of dimension $J \times J$ (Hosmer and Lemeshow, 2000:168). The leverage values are the diagonal elements, h_j, of the hat matrix. Instead of plotting the leverage values versus the fitted values, it is more useful to plot the fitted values against three different measures.

The standardized Pearson residual is central to each of the three measures. Recalling the Pearson residual from equation (2.19), the standardized Pearson residual for the j^{th} distinct covariate value is

$$r_{sj} = \frac{r_j}{\sqrt{1-h_j}}, \quad \text{for } j = 1, 2, \ldots, J\ . \qquad (2.25)$$

A useful measure resulting from equation (2.25) is the standardized difference between $\hat{\boldsymbol{\beta}}$ and $\hat{\boldsymbol{\beta}}_{(-j)}$, where $\hat{\boldsymbol{\beta}}_{(-j)}$ is the maximum likelihood estimates of the model coefficients with the m_j observations for the j^{th} distinct covariate value removed. This measure, denoted $\Delta\hat{\boldsymbol{\beta}}_j$, is expressed as

$$\Delta\hat{\boldsymbol{\beta}}_j = \left(\hat{\boldsymbol{\beta}} - \hat{\boldsymbol{\beta}}_{(-j)}\right)^T \left(\mathbf{X}^T\mathbf{V}\mathbf{X}\right)\left(\hat{\boldsymbol{\beta}} - \hat{\boldsymbol{\beta}}_{(-j)}\right)$$

$$= \frac{r_{sj}^2 h_j}{\left(1-h_j\right)} \qquad (2.26)$$

(Hosmer and Lemeshow, 2000:173). Letting d_j be the deviance of the model with the m_j observations for the j^{th} distinct covariate value removed, the difference in deviance, ΔD_j, is given by

$$\Delta D_j = d_j^2 + \frac{r_j^2 h_j}{\left(1-h_j\right)} \qquad (2.27)$$

The change in the value of the Pearson chi-square statistic is shown to be equal to the square of the standardized Pearson residual of equation (2.25).

$$\Delta X_j^2 = \frac{r_j^2}{\left(1-h_j\right)} = r_{sj}^2 \qquad (2.28)$$

Distinct covariate values that are inadequately fitted can be identified by large values of ΔD_j, ΔX_j^2, or both. Large values of $\Delta\hat{\boldsymbol{\beta}}_j$ indicate influence points. That is, distinct covariate values that exert a significant amount of influence on the estimated values of the model coefficients (Hosmer and Lemeshow, 2000:174).

Testing Significance of Individual Coefficients.

The likelihood ratio test, *G*, is a test of the hypothesis that all of the model coefficients are zero. It is statistically equivalent to the global F test in linear regression. The Wald test, *W*, is statistically equivalent to the partial F test in linear regression. It

assesses the individual significance of the j^{th} covariate. The null and alternative hypotheses for the j^{th} coefficient are given by

$$\begin{aligned} H_0 &: \beta_j = 0 \\ H_A &: \beta_j \neq 0 \end{aligned} \tag{2.29}$$

For a multivariate model, *G* can be computed by subtracting the deviance of the model containing the j^{th} variable from the deviance of the model that does not contain the j^{th} covariate. Because the likelihood for the saturated model is included in both deviance calculations, *G* is typically expressed as two times the natural log of the likelihood ratio between the model containing the j^{th} covariate and the model that does not contain the j^{th} covariate.

In the univariate case, the expected value, or probability of success, of the model that does not contain the single covariate becomes a simple proportion, or the ratio of the frequency of observations where $y = 1$ to the total number of observations *n*. Similarly, the probability of failure becomes a ratio of the frequency of observations where $y = 0$ to the total number of observations. Thus, the likelihood function for the model that does not contain the covariate is $\left(n_1/n\right)^{n_1}\left(n_0/n\right)^{n_0}$, where $n_1 = \sum y_j$, $n_0 = \sum\left(1 - y_j\right)$, and $y_j = 1$. The likelihood ratio test statistic *G* then becomes

$$G = 2\ln\left(\frac{\prod_{j=1}^{n}\hat{\pi}_j^{y_j}\left(1-\hat{\pi}_j\right)^{1-y_j}}{\left(\frac{n_1}{n}\right)^{n_1}\left(\frac{n_0}{n}\right)^{n_0}}\right) \tag{2.30}$$

Further simplifying equation (2.30) yields an expression in which the outputs from MINITAB can easily be substituted.

$$G = 2\left(\sum_{j=1}^{n}\left(y_j \ln\left(\hat{\pi}_j\right) + \left(1 - y_j\right)\ln\left(1 - \hat{\pi}_j\right)\right) - \left(n_1 \ln\left(n_1\right) + n_0 \ln\left(n_0\right) - n \ln\left(n\right)\right)\right) \qquad (2.31)$$

Since this is a test for the significance of a covariate, rather than a test for model adequacy, the test statistic G is distributed chi-square with one degree of freedom. The rejection region criteria are

$G \le \chi^2_{\alpha,1}$, fail to reject H_0, or

$G > \chi^2_{\alpha,1}$, covariate is significant.

For multivariate models, rejection of the null hypothesis implies that *at least* one of the covariates is significant. Additional hypothesis tests are needed to determine which one(s). One might also use the p-value approach to evaluate the significance of a covariate. That is, if $P\left(\chi^2_1 > G\right) < \alpha$, then sufficient evidence exists to imply the significance of the covariate under test (Hosmer and Lemeshow, 2000:14-15).

The *Hessian* matrix, or the $(k+1)\times(k+1)$ matrix of second partial derivatives of equation (2.14), is derived to support the Wald test. The quantities of interest are the diagonal elements of the negative inverse of the Hessian, which are evaluated at the maximum likelihood estimators $\hat{\boldsymbol{\beta}}$. The square roots of these diagonal elements are the standard errors of the coefficients of equation (2.4), which MINITAB computes automatically. The Wald test statistic, *W*, under the null hypothesis in equation (2.29) is

$$W = \frac{\hat{\beta}_j}{se\left(\hat{\beta}_j\right)} \qquad (2.32)$$

where $se\left(\hat{\beta}_j\right)$ denotes the standard error of the j^{th} regression coefficient. Two methods can be used to compare *W*, but MINITAB uses a p-value approach. The Wald statistic

can be squared and compared to a chi-square distribution with one degree of freedom, as with the likelihood ratio test. MINITAB examines a probability taken from the standard normal distribution. That is, if $P\left(|z| > W\right) < \alpha$, then the j^{th} covariate can be said to contribute significantly to the model (Montgomery, Peck, and Vining, 2001:458).

Confidence intervals (CIs) on both the estimated model parameters and the odds ratios can be computed. A CI provides a degree of assurance about the accuracy of a maximum likelihood estimate (MLE). The narrower the range of the CI, then the higher the confidence is that the MLE closely approximates the true parameter value. MINITAB, however, only outputs CIs for the estimated odds ratios. Consequently, only the procedures for constructing CIs on odds ratios are described here, but inferences for CIs on the model coefficients can easily be made. MINITAB constructs 95% CIs by default. Thus, at the $\alpha = 0.05$ level of significance, a 95% CI on the j^{th} odds ratio is expressed as

$$\exp\left(\hat{\beta}_j \pm z_{1-0.05/2} \times se\left(\hat{\beta}_j\right)\right) \tag{2.33}$$

(Hosmer and Lemeshow, 2000:52-53).

<u>Multinomial Logistic Regression.</u>

When the focus of a war termination study is placed on the methods by which wars end, rather than on the winners and losers of wars, examination of Pillar’s analyses alone show the response variable of interest to contain more than two categories, or methods of ending wars. Hence, binary logistic regression cannot be used to analyze this situation because the response is polychotomous, rather than dichotomous. Modifications

to the binary logistic regression model were made in 1974, and the result was the multinomial logistic regression model (Hosmer and Lemeshow, 2000:260). The term *multinomial* is used because the outcome variable, or type of war ending, is said to be *nominal*. This follows from the fact that types of war endings cannot be ordered in any statistically meaningful way (Hosmer and Lemeshow, 2000:260).

The simplest way to demonstrate the theory behind multinomial logistic regression is through the case where the response contains $p=3$ categories, though extensions of the model can easily be made for responses containing more than three categories. Let the categories of the response variable, *Y*, be coded as 0, 1, and 2. MINITAB, however, allows the response code to begin with 1, rather than 0. For any response with *p* categories, a reference category must be selected, to which the remaining $p-1$ categories are compared. The $Y=0$ category is selected as the reference category for explaining multinomial logistic regression theory here, which is the same assumption made by Hosmer and Lemeshow (Hosmer and Lemeshow, 2000:261).

While binary logistic regression makes use of only one logit function, multinomial logistic regression produces $p-1$ logits. Each logit is expressed as the natural logarithm of a ratio of conditional probabilities. In general, the conditional probability for the j^{th} response category given $\mathbf{x}$, where $\mathbf{x}$ is a vector of *k* covariates plus a constant term, is given by

$$P(Y=j\,|\,\mathbf{x})=\frac{\exp\left(g_j(\mathbf{x})\right)}{1+\sum_{i=1}^{p-1}\exp\left(g_i(\mathbf{x})\right)}=\pi_j(\mathbf{x}) \qquad (2.34)$$

The j^{th} logit, for which MLEs are computed, is denoted as

$$g_j(\mathbf{x}) = \ln\left(\frac{P(Y=j\,|\,\mathbf{x})}{P(Y=0\,|\,\mathbf{x})}\right)$$

$$= \beta_{j0} + \beta_{j1}x_1 + \beta_{j2}x_2 + \cdots + \beta_{jk}x_k$$

$$= \mathbf{x}^T \boldsymbol{\beta}_j \tag{2.35}$$

where $j = 1, 2, \ldots, p-1$. It follows that the logit for $Y = i$ versus $Y = j$ can be computed by

$$g_{i,j}(\mathbf{x}) = g_i(\mathbf{x}) - g_j(\mathbf{x})$$

$$= \ln\left(\frac{P(Y=i\,|\,\mathbf{x})}{P(Y=0\,|\,\mathbf{x})}\right) - \ln\left(\frac{P(Y=j\,|\,\mathbf{x})}{P(Y=0\,|\,\mathbf{x})}\right)$$

$$= \ln\left(\frac{P(Y=i\,|\,\mathbf{x})}{P(Y=0\,|\,\mathbf{x})}\frac{P(Y=0\,|\,\mathbf{x})}{P(Y=j\,|\,\mathbf{x})}\right)$$

$$= \ln\left(\frac{P(Y=i\,|\,\mathbf{x})}{P(Y=j\,|\,\mathbf{x})}\right)$$

$$= \mathbf{x}^T\left(\boldsymbol{\beta}_i - \boldsymbol{\beta}_j\right). \tag{2.36}$$

For the purpose of clarifying the likelihood function, the response is coded using indicator, or dummy, variables (Montgomery, Peck, and Vining, 2001:265). A *p*-category response can be coded using p dummy variables as follows:

If $Y = 0$, then $v_0 = 1$, $v_1 = 0$, $v_2 = 0, \ldots, v_{p-1} = 0$.

If $Y = 1$, then $v_0 = 0$, $v_1 = 1$, $v_2 = 0, \ldots, v_{p-1} = 0$.

If $Y = 2$, then $v_0 = 0$, $v_1 = 0$, $v_2 = 1, \ldots, v_{p-1} = 0$.

$\vdots$

If $Y = p-1$, then $v_0 = 0$, $v_1 = 0$, $v_2 = 0, \ldots, v_{p-1} = 1$.

$$\sum_{j=0}^{p-1} v_j = 1 \text{ for any } i = 1, 2, \ldots, n.$$

Letting π_{ji} denote the j^{th} conditional probability function corresponding to the response from the i^{th} observation, and letting g_{ji} denote the j^{th} logit corresponding to the response from the i^{th} observation, the conditional likelihood function takes the form

$$l(\boldsymbol{\beta}) = \prod_{i=1}^{n} \left(\pi_{0i}^{v_0} \pi_{1i}^{v_1} \pi_{2i}^{v_2} \cdots \pi_{(p-1)i}^{v_{p-1}} \right).$$

It follows that the log-likelihood function is

$$L(\boldsymbol{\beta}) = \sum_{i=1}^{n} v_{1i} g_{1i} + v_{2i} g_{2i} + \cdots + v_{(p-1)i} g_{(p-1)i} - \ln\left(1 + e^{g_{1i}} + e^{g_{2i}} + \cdots + e^{g_{(p-1)i}}\right). \quad (2.37)$$

Taking first partial derivatives yields $(p-1)(k+1)$ likelihood equations. This result is shown by noting that a p-category response produces $p-1$ logits, each containing $k+1$ parameters. As with binary logistic regression, setting the likelihood equations to zero and solving for $\boldsymbol{\beta}$ gives the MLEs, $\hat{\boldsymbol{\beta}}$, which are again obtained via the IRLS procedure (Hosmer and Lemeshow, 2000:262-263).

Interpretation of the parameters is similar to that of the binary model. There are $k(p-1)$ odds ratios to compute, in which each of the remaining $p-1$ response values is compared to the reference category. It is assumed here that the reference outcome is $Y = 0$, but MINITAB allows the selection of any category as the reference. For a continuous covariate, the odds ratio comparing $Y = j$ to $Y = 0$ associated with a one-unit change in x, is expressed as

$$\hat{O}_{Rj} = \frac{P(Y = j \mid X = x) \big/ P(Y = 0 \mid X = x)}{P(Y = j \mid X = x \pm 1) \big/ P(Y = 0 \mid X = x \pm 1)} \tag{2.38}$$

(Hosmer and Lemeshow, 2000:265).

Calculations for the likelihood ratio statistic are similar to those for the binary logistic regression model. The difference lies in the degrees of freedom associated with it. For a continuous covariate, the likelihood ratio statistic, *G*, is distributed chi-square with $p-1$ degrees of freedom. For a categorical covariate, also called a factor, the degrees of freedom become $(p_r - 1)(p_f - 1)$, where p_r is the number of categories in the response, and p_f is the number of categories in the factor (Hosmer and Lemeshow, 2000:270).

Hosmer and Lemeshow note that ideas for extending diagnostic measures into multinomial models have been proposed. Current statistical software packages, however, have not incorporated such proposals because the measures involved are computationally intensive (Hosmer and Lemeshow, 2000:281). As a result, diagnostic measures and plots were not generated for the multinomial models on interstate wars in this study. The odds ratios, goodness-of-fit tests, and likelihood ratio tests were considered sufficient to achieve the overarching goal of demonstrating the applicability of multinomial logistic regression to war termination investigations.

Summary

This thesis seeks to define probabilistic relationships between the outcomes or winners of wars and a single or group of explanatory variables. Constructing the best descriptive and most parsimonious models from the available open-source data is also desired. Logistic regression techniques provide readily interpretable ways of defining such relationships. Because war is a complex endeavor and the conduct of war is highly dynamic, the termination of war is described best through conditional probabilities and likelihoods. The results of logistic regression can also provide additional insights into what levels of which explanatory variables are either necessary or acceptable in order to either achieve a particular war ending or emerge victorious from a war.

III. Methodology

Rationale

The goal of this research is to investigate and define, if possible, relationships between several independent variables and either the winner of a war or the manner in which a war ends. Given the qualitative nature of the dependent variables of the selected data sets, a logistic regression approach is the preferred method to model such relationships. The dependent variable is commonly called the response, and the independent variables are called covariates (Hosmer and Lemeshow, 2000:1).

Unlike linear regression, the response for each data set is categorical. For the interstate wars set, the response is denoted by the variable Outcome. For both the intrastate wars and extra-state wars sets, the response is denoted by the variable Winner. Each of the response variables is nominal. That is, no natural ordering of its categories exists, and numerical differences between categories are meaningless. Each response contains six categories, so the resulting model is called a polychotomous or multinomial logistic regression model (Hosmer and Lemeshow, 2000:260). The term *multinomial* is preferred in this thesis.

Variable Selection

The data set concerning participants in interstate wars initially contained 28 variables. These variables and their COWP definitions are given in Appendix A. The COWP assigned a unique number to each participant, called a country code, so it was

assumed that neither the country code nor the three-letter country abbreviation needed be included in the final data set. The initial set also contained variables for the days, months, and years in which the individual wars began and ended. The COWP included a second set of date columns for those wars in which there was a short break in the fighting, but the war started up again. Existing war termination literature does not appear to emphasize the importance of dates. It was therefore believed that these variables were unnecessary for the analysis, so the date columns were not added to the final data set. A similar assumption was made about the variables concerning the geographic location of the wars, although this may be an area for future investigation. Ultimately, five variables were retained for analysis: the outcome of the war for the participating nation, the duration of the war in days, the participating nation's population at the war's outset, the participating nation's military manpower at the war's outset, and the number of combat deaths sustained during the war by the participating nation. Identical assumptions were made for both the extra-state and intrastate war sets, and the same five variables were retained. However, the response variable was defined by who won the conflict, rather than how the conflict ended.

Variable Translation

Any nation, past or present, has or has had the potential to engage in armed conflict. Some nations are small, and some are considered superpowers. Therefore, it is not sufficient to analyze the raw data. Measures that adequately describe the entire population of belligerents are needed. Expressing the casualty, population, and armed forces data as proportions was believed to yield more meaningful and interpretable results

than the raw numbers. Three proportions were computed for each observation in each data set,

$$\%\_Casualties = \frac{C\_Deaths}{Tot\_Deaths} \tag{3.1}$$

$$Deaths/Pop\_\% = \frac{C\_Deaths}{PWarPop} \tag{3.2}$$

$$Deaths/Arm\_\% = \frac{C\_Deaths}{PWarArm} \tag{3.3}$$

where *C_Deaths* is the number of casualties sustained by the participant during the war, *Tot_Deaths* is the sum of casualties sustained by all belligerents during the war, *PWarPop* is the participating nation's population at the start of the war, and *PWarArm* is the size of the participating nation's armed forces at the start of the war.

An attempt was made to create a proxy measure of the economic costs of wars and include such a measure in the multinomial logistic regression model. This proxy measure was derived from other data sets compiled by the COWP. In their National Material Capabilities (NMC) data set, the COWP included yearly observations of military expenditures, in millions of 2001 US dollars (USD). The variables for this set and their definitions are given in Appendix C.

For each war participant, the average amount of military expenditures, denoted as *Avg_Milex*, was computed for the duration of each war. The desire was to take that average and divide it by the average gross domestic product (GDP) for each war participant during each war, which would have given a proxy measure for the degree to which a nation's industrial capacity is consumed by war. Unfortunately, GDP figures could not be obtained for wars occurring earlier than 1870, and, of the GDP estimates

available, not enough countries contained GDP observations to cover the number of participants in the interstate wars data set. It should be noted that while GDP figures might be obtained from other sources, one secondary objective of this study was only to use data from the same open source, the COWP. As a result, another more available proxy economic indicator was used. The COWP, in its data set on national trade, compiled total trade estimates for each of the countries in the interstate wars set.

$$avgME\_as\_PTT = \frac{Avg\_Milex}{Avg\_TTrade} \tag{3.4}$$

The COWP computed total trade as a sum of a nation's total imports and total exports for a given year, all in 2001 USD. For each war participant, the average total trade, *Avg_TTrade*, was computed for the duration of each war, and this amount was used as the divisor in lieu of average GDP. This proxy measure was defined as the average amount of military spending as a proportion of the average total trade for the war. Without consistent GDP estimates, this measure was proposed as the best economic activity indicator available for this analysis.

The category definitions for the response *Outcome* in the interstate wars data set were revised from those given by the COWP, which are given in Table 1. Determining the likelihood of one type of outcome over another was assumed to be more important to this study than knowing on which side a given country participated, so the new definitions were created by comparing the COWP definitions to those of Paul Pillar's classifications. The revised response categories for the interstate wars data are given in Table 2. In contrast, the response categories for the intrastate and extra-state sets did not require revision, and the next section explains this case.

Table 1: COWP Outcomes for Interstate Wars

Category	COWP Definition
1	On Winning Side
2	On Losing Side
3	On Side A of a Tie
4	On Side B of a Tie
5	On Side A of an Ongoing War
6	On Side B of an Ongoing War

For the cases where either a total military conquest, which Pillar calls extermination or expulsion, or an imposed settlement ends a war, it was assumed that the victor's military force was the dominant factor. That is, the winning side inflicts military defeats upon his enemy to such an extent that his enemy must give up the fight through either unconditional surrender or capitulation to terms imposed upon him during an armistice or cease-fire. These cases were subsequently defined, and thus categorized, as victory through military imposition (Pillar, 1983:14).

The converse of the aforementioned definition was assumed to be true when considering a martially defeated nation. The losing country agrees to the demands of the victor, no matter in what manner such an agreement occurs. Pillar's description of this type of situation was considered accurate, so this category was called capitulation (Pillar, 1983:15).

Defining the cases where no clear victor exists, or where a clear military victor emerges without the capitulation of the defeated, is difficult. Pillar refers to a mutual withdrawal of military forces, either with or without an agreement (Pillar, 1983:14). However, in order to distinguish from a negotiation, it is assumed that fighting ceases without any resolution of the issues over which the war was waged. The circumstances surrounding some cease-fires and armistices may cause them to fall into this category,

such as those of the cease-fires between Israel and one or more of the Arab states in 1949, 1956, 1967, 1973, 1982, and 2006 (Pillar, 1983:22-23). These cases constitute stalemates.

Another difficulty arose with the few observations where the participants began fighting a small war, but either the conflict grew into a major war through third-party intervention, or the participants joined allies in a larger war to fight for different aims. Pillar calls this absorption (Pillar, 1983:14). Because there were so few of these cases in the data set, each observation exhibiting this result was examined to find conditions that would allow it to be placed in a previously defined category. Such conditions existed in some of the observations, but not in all. Since the sample size for the interstate wars set was larger than 200 observations, it was assumed that the two observations fitting the aforementioned description would inflate the range of the CIs for the resulting odds ratios, so the two observations were omitted from the data set.

When imposition, capitulation, or a stalemate does not occur, then the possibility exists for a mutual agreement between all belligerents to occur. Such an agreement is not one-sided, but rather all sides make concessions in order to form a pact about which all can be satisfied. In such situations of compromise, it is assumed that some form of negotiation between opposing nations must take place (Pillar, 1983:15). Unlike Pillar, who makes a distinction between agreements between belligerents and third-party mediations, the fact that a compromise is struck is assumed to be more important than the manner in which it is struck.

The COWP also compiled a data set concerning international disputes, called Militarized Interstate Disputes (MID). The variables for the MID set and their COWP

definitions are given in Appendix B. The subset of the MID set where the disputes resulted in wars matched exactly to the observations in the interstate wars set. The advantage to this was that the values for the outcome and settlement variables in the MID subset could be directly compared to the corresponding values for the response in the interstate wars set. The purpose of this comparison was to distinguish between those participants who benefited the most, or won, through a negotiated settlement and those participants who gained the least, or lost, through a negotiated settlement. That is, those observations whose MID outcome was a compromise and settlement was negotiated, but whose interstate wars outcome was a victory, are coded under the category of victory by negotiated settlement. Those observations whose MID outcome was a compromise and settlement was negotiated, but whose interstate wars outcome was a yield, are coded under the category of defeat by negotiated settlement.

Table 2: Revised Outcomes for Interstate Wars

Category	Revised Definition
1	Victory by Military Imposition
2	Capitulation
3	Stalemate
4	Victory by Negotiated Settlement
5	Defeat by Negotiated Settlement

Data Compression

The next obstacle was to deal with any missing data for each set. Each variable had missing entries, but not all of the missing entries occurred in the same observation. Several statistical techniques could have been used to fill in the missing entries, but the sample sizes for each set remained sufficiently large with the observations corresponding

to the missing entries omitted. The rule of 10, as discussed by Hosmer and Lemeshow, was used to justify eliminating the missing data points from the final sets (Hosmer and Lemeshow, 2000:346-347).

The objective of the rule of 10 is to determine the number of observations per estimated parameter needed to avoid poor model variance estimates. Reviewing the observations per parameter also allows the flexibility to postulate higher-order models, as opposed to main effects models only. Hosmer and Lemeshow use the quantity $m = \min(n_1, n_0)$, where n_1 and n_0 are the frequencies of those observations yielding responses of 1 and 0, respectively. However, the above quantity is assuming the use of a typical dichotomous, or binomial, logistic regression model, where the outcome can only assume one of two values (Hosmer and Lemeshow, 2000:346).

The response *Outcome* in the interstate wars set contains five categories, so the quantity used by Hosmer and Lemeshow is revised to reflect a multinomial logistic regression model.

$$m = \min(n_0, n_1, n_2, n_3, n_4) \tag{3.5}$$

For equation (3.5), n_0 is the number of wars where the participant wins by military imposition, n_1 is the number of wars where the participant loses through capitulation, n_2 is the number of wars ending by stalemate, n_3 is the number of wars where the participant wins through a negotiated settlement, and n_4 is the number of wars where the participant loses through a negotiated settlement. After eliminating the observations containing missing data, 225 observations remained. The least frequent response was $m = \min(87, 53, 31, 26, 28) = 26$, or a victory through a negotiated settlement. For k covariates, Hosmer and Lemeshow suggest that $k + 1 \leq m/10$ parameters be included in

the model, where $k+1$ is the number of covariates plus an intercept term (Hosmer and Lemeshow, 2000:346). No more than $26/10 = 2.6 \approx 2$ parameters should be included in the interstate wars model, which corresponds to a univariate, or single-variable main effects, model.

For both the extra-state and intrastate wars sets, when the observations containing missing data were eliminated, their respective response categories reduced to the binomial case. That is, the remaining response values corresponded to either the state winning or the non-state actor or insurgency winning. Table 3 shows the resulting categories and definitions for both the extra-state and intrastate data sets.

Table 3: Winner Categories for Extra/Intrastate Wars

Category	Definition
1	State Wins
2	Non-State Actor/Insurgency Wins

Let m_1 be the smaller frequency for the intrastate data set, and let m_2 be the smaller frequency for the extra-state data set. For the intrastate wars, $m_1 = \min(49, 24) = 24$, so the model should contain no more than $24/10 = 2.4 \approx 2$ parameters, which again corresponds to a univariate main-effects model. For the extra-state wars, $m_2 = \min(40,19) = 19$, so its model should have $19/10 = 1.9 \approx 1$ parameter, which would exclude any covariates and contain only a constant term.

It should be noted that the rule of 10 is not absolute. Hosmer and Lemeshow insist that it be used as a guideline only. Other considerations must be made, such as the balance of the distribution of the covariates, total sample size, and any previously stated

requirements. If the distribution of multinomial response is skewed towards one category or a subset of its categories, then the applicability of the rule of 10 could be questionable (Hosmer and Lemeshow, 2000:347). Skewed response variables were present in each of the three data sets analyzed. Therefore, first-order main-effects models including all retained covariates were postulated initially for each data set such that the usefulness of the rule of 10, at least in this case, could be determined.

Unit Normal Scaling

Unit normal data scaling was used to aid in interpretation of the odds ratios for the fitted models. Unlike the responses, the covariates, once translated into proportions, were continuous, so it was assumed that each was approximately normally distributed. The idea of a single-unit change in each covariate needed to be defined as well. Unit normal scaling provided these definitions.

This technique involves transforming a normal random variable into a standard normal random variable. For $i = 1, 2, \ldots, n$; and for $j = 1, 2, \ldots, k$; the i^{th} observation of the j^{th} covariate is expressed as

$$z_{ij} = \frac{x_{ij} - \overline{x}_j}{s_j} \tag{3.6}$$

where the sample variance of the j^{th} covariate is given by

$$s_j^2 = \frac{\sum_{i=1}^{n} \left(x_{ij} - \overline{x}_j \right)^2}{n-1}$$

and the sample mean of the j^{th} covariate is

$$\bar{x}_j = \frac{1}{n}\sum_{i=1}^{n} x_{ij} \, .$$

As with the standard normal distribution, each scaled covariate has a mean of 0 and a standard deviation of 1 (Montgomery, Peck, and Vining, 2001:113).

Trend Recognition

The observations in each of the three data sets analyzed covered nearly two centuries of warfare, from as early as 1816 to as late as 1997. In addition to the obvious improvements in weapons and subsequent shifts in tactics, the question of whether or not similar shifts in war termination patterns could be found was addressed. In order to identify such pattern changes, subsets of each data set needed to be analyzed, which prompted another question. How should the data be divided?

Two methods of data division were considered. Since the data covered two centuries of war, a proposed dividing line was the year 1900. That is, all observations occurring before 1900 would be used to fit one model, while all observations occurring in 1900 and after would be used to fit a separate model. This division method could account for weapons technology changes between the nineteenth and twentieth centuries. Dividing the data by major shifts in tactics, such as the switch from Napoleonic-style combat to smaller squad-level maneuvers, was another proposal. Typically, though not immediately, improvements in weapons technology necessarily prompt changes in how weapons are employed in war. While certainly open to historical debate, the Spanish-American War of 1898 was assumed to be the transition point from Napoleonic warfare

to modern, or mechanized, warfare. Ultimately, the composition of the data sets allowed divisions such that both changes in century and changes in tactics could be simultaneously examined.

Using the above method, MINITAB was used to fit three logistic regression models to each of the three war data sets. Multinomial logistic regression was employed for the interstate wars set, where the response *Outcome* contained five categories. Compressing and translating the data from both the intrastate and extra-state sets allowed the use of binary logistic regression, with *Winner* as the response in both cases. The first models for each set were fit using the aggregated data in each set. The second models were fit using the divided data from the 19th Century, while the last models used the divided data from the 20th Century.

Variable Nomenclature

Different names were given to each response and covariate for each data set. The variable names in each set included a designator for the data scaling technique used, unit normal scaling (UNS). The variables names were additionally distinguished by century. The variable names in the aggregated models, however, did not contain century designators. Table 4 contains each response variable name included in this study and its corresponding definition. The names and definitions for the extra-systemic war covariates used in this study are shown in Table 5. The names and definitions for the intrastate war covariates used in this study are shown in Table 6. The names and definitions for the interstate war covariates used in this study are given in Table 7.

Table 4: Response Variable Nomenclature

Response	Definition
Winner_ES_UNS_19	Extra-systemic War Winner, 19th Century Wars
Winner_ES_UNS_20	Extra-systemic War Winner, 20th Century Wars
Winner_ES_UNS	Extra-systemic War Winner, Aggregated Wars
Winner_IS_UNS_19	Intrastate War Winner, 19th Century Wars
Winner_IS_UNS_20	Intrastate War Winner, 20th Century Wars
Winner_IS_UNS	Intrastate War Winner, Aggregated Wars
Outcome(PR2)_19	Outcome of Interstate War, 19th Century Wars
Outcome(PR2)_20	Outcome of Interstate War, 20th Century Wars
Outcome(PR2)	Outcome of Interstate War, Aggregated Wars

Table 5: Covariate Nomenclature for Extra-Systemic Wars

Covariate	Definition
Dur_ES_UNS_19	Duration of 19th Century Extra-Systemic War, Unit Normally Scaled
C_Dths/Pop_ES_UNS_19	Proportion of State Deaths to Its Pre-war Population, 19th Century Extra-Systemic Wars, Unit Normally Scaled
C_Dths/Arm_ES_UNS_19	Proportion of State Deaths to Its Pre-war Armed Force Size, 19th Century Extra-Systemic Wars, Unit Normally Scaled
C_Dths/TDths_ES_UNS_19	Proportion of Total Deaths Sustained by Participant, 19th Century Extra-Systemic Wars, Unit Normally Scaled
Dur_ES_UNS_20	Duration of 20th Century Extra-Systemic War, Unit Normally Scaled
C_Dths/Pop_ES_UNS_20	Proportion of State Deaths to Its Pre-war Population, 20th Century Extra-Systemic Wars, Unit Normally Scaled
C_Dths/Arm_ES_UNS_20	Proportion of State Deaths to Its Pre-war Armed Force Size, 20th Century Extra-Systemic Wars, Unit Normally Scaled
C_Dths/TDths_ES_UNS_20	Proportion of Total Deaths Sustained by Participant, 20th Century Extra-Systemic Wars, Unit Normally Scaled
C_Deaths/Pop_ES_UNS	Proportion of State Deaths to Its Pre-war Population, Aggregated Extra-Systemic Wars, Unit Normally Scaled
C_Deaths/Arm_ES_UNS	Proportion of State Deaths to Its Pre-war Armed Force Size, Aggregated Extra-Systemic Wars, Unit Normally Scaled
C_Deaths/TotDeaths_ES_UNS	Proportion of Total Deaths Sustained by Participant, Aggregated Extra-Systemic Wars, Unit Normally Scaled
Duration_ES_UNS	Duration of Aggregated Extra-Systemic Wars, Unit Normally Scaled

Table 6: Covariate Nomenclature for Intrastate Wars

Covariate	Definition
Duration_IS_UNS_19	Duration of 19th Century Intrastate War, Unit Normally Scaled
Dead/Pop_IS_UNS_19	Proportion of State Deaths to Its Pre-war Population, 19th Century Intrastate Wars, Unit Normally Scaled
Dead/Arm_IS_UNS_19	Proportion of State Deaths to Its Pre-war Armed Force Size, 19th Century Intrastate Wars, Unit Normally Scaled
C_Dead/TotDead_IS_UNS_19	Proportion of Total Deaths Sustained by Participant, 19th Century Intrastate Wars, Unit Normally Scaled
Duration_IS_UNS_20	Duration of 20th Century Intrastate War, Unit Normally Scaled
Dead/Pop_IS_UNS_20	Proportion of State Deaths to Its Pre-war Population, 20th Century Intrastate Wars, Unit Normally Scaled
Dead/Arm_IS_UNS_20	Proportion of State Deaths to Its Pre-war Armed Force Size, 20th Century Intrastate Wars, Unit Normally Scaled
C_Dead/TotDead_IS_UNS_20	Proportion of Total Deaths Sustained by Participant, 20th Century Intrastate Wars, Unit Normally Scaled
Duration_IntS_UNS	Proportion of State Deaths to Its Pre-war Population, Aggregated Intrastate Wars, Unit Normally Scaled
Dead/Pop_IntS_UNS	Proportion of State Deaths to Its Pre-war Armed Force Size, Aggregated Intrastate Wars, Unit Normally Scaled
Dead/Arm_IntS_UNS	Proportion of Total Deaths Sustained by Participant, Aggregated Intrastate Wars, Unit Normally Scaled
C_Dead/TotDead_IntS_UNS	Duration of Aggregated Intrastate Wars, Unit Normally Scaled

Table 7: Covariate Nomenclature for Interstate Wars

Covariate	Definition
Duration_UNS_19	Duration of 19th Century Interstate War, Unit Normally Scaled
Dths/Pop_UNS_19	Proportion of State Deaths to Its Pre-war Population, 19th Century Interstate Wars, Unit Normally Scaled
Dths/Arm_UNS_19	Proportion of State Deaths to Its Pre-war Armed Force Size, 19th Century Interstate Wars, Unit Normally Scaled
MilEx/TT_UNS_19	Proportion of Average State Military Expenditures (2001 USD) to Average State Total Trade (2001 USD), 19th Century Interstate Wars, Unit Normally Scaled
Dths/TDeaths_UNS_19	Proportion of Total Deaths Sustained by Participant, 19th Century Interstate Wars, Unit Normally Scaled
Duration_UNS_20	Duration of 20th Century Interstate War, Unit Normally Scaled
Dths/Pop_UNS_20	Proportion of State Deaths to Its Pre-war Population, 20th Century Interstate Wars, Unit Normally Scaled
Dths/Arm_UNS_20	Proportion of State Deaths to Its Pre-war Armed Force Size, 20th Century Interstate Wars, Unit Normally Scaled
MilEx/TT_UNS_20	Proportion of Average State Military Expenditures (2001 USD) to Average State Total Trade (2001 USD), 20th Century Interstate Wars, Unit Normally Scaled
Dths/TDeaths_UNS_20	Proportion of Total Deaths Sustained by Participant, 20th Century Interstate Wars, Unit Normally Scaled
Duration_UNS	Duration of Aggregated Interstate Wars, Unit Normally Scaled
Deaths/Pop_UNS	Proportion of State Deaths to Its Pre-war Population, Aggregated Interstate Wars, Unit Normally Scaled
Deaths/Arm_UNS	Proportion of State Deaths to Its Pre-war Armed Force Size, Aggregated Interstate Wars, Unit Normally Scaled
Deaths/TotDeaths_UNS	Proportion of Total Deaths Sustained by Participant, Aggregated Interstate Wars, Unit Normally Scaled
MilEx/TotTrade_UNS	Proportion of Average State Military Expenditures (2001 USD) to Average State Total Trade (2001 USD), Aggregated Interstate Wars, Unit Normally Scaled

Stepwise Regression

Stepwise regression is a robust procedure commonly used in both linear and logistic regression as a model-building technique. This is an effective technique to use when the true relationship between the covariates and the response is either unknown or unclear (Hosmer and Lemeshow, 2000:116). Stepwise regression was employed for this research because, as an initial investigation, the associations within the COWP data were unknown. They were also unclear in the sense that war termination literature has identified several factors, some of which were included in the COWP data, as directly related to the outcome of a conflict, but the extent to which such factors were statistically relevant had not previously been determined.

As noted in the previous chapter, significance of a covariate in logistic regression is identified by the likelihood ratio test. Thus, the most significant covariate is the one with the largest likelihood ratio statistic, *G* (Hosmer and Lemeshow, 2000:116). The stepwise procedure begins with a pool of *k* covariates. The covariates can be either categorical or continuous, but because the covariates for this research are continuous, the notation presented here reflects that used for continuous covariates only. Stepwise regression for logistic models is described here as a series of steps.

Step 0: Fit a constant only model. Let L_0 be the log-likelihood value for the constant only model. Estimate *k* univariate logistic regression models, one for each covariate in the pool. Let $L_j^{(0)}$ be the log-likelihood value for the model containing the j^{th} covariate in Step 0, where $j = 1, 2, \ldots, k$. Using equation (2.31), the likelihood ratio test is expressed as

$$G_j^{(0)} = 2\left(L_j^{(0)} - L_0\right). \tag{3.7}$$

Let the p-value for the j^{th} likelihood ratio statistic be

$$P\left(\chi^2_{\alpha,1} > G_j^{(0)}\right) = p_j^{(0)}. \tag{3.8}$$

Since the most significant covariate is that with the largest likelihood ratio statistic, then the covariate with the smallest p-value yields the same conclusion. Let

$$p_{e_1}^{(0)} = \min\left(p_1^{(0)}, p_2^{(0)}, \ldots, p_k^{(0)}\right), \tag{3.9}$$

where $p_{e_1}^{(0)}$ denotes the p-value corresponding to the covariate selected to enter the model at Step 1, provided that the value does not equal or exceed the p-value corresponding to a previously defined significance level (Hosmer and Lemeshow, 2000:117). Let p_E be the p-value for entry such that $p_{e_1}^{(0)} < p_E$. If $p_{e_1}^{(0)} \geq p_E$, then end the procedure because no covariates enter the model. Otherwise, let x_{e_1} denote the covariate corresponding to the minimum p-value, $p_{e_1}^{(0)}$, and go to Step 1 (Hosmer and Lemeshow, 2000:118).

Step 1: Estimate the logistic regression model containing x_{e_1}, and let $L_{e_1}^{(1)}$ be the resulting log-likelihood of the model. Estimate $k-1$ models that contain both x_{e_1} and x_j, where $j = 1, 2, \ldots, k$ and $j \neq e_1$. For each of these $k-1$ models, let $L_{e_1,j}^{(1)}$ denote its log-likelihood value. The j^{th} likelihood ratio statistic becomes

$$G_j^{(1)} = 2\left(L_{e_1,j}^{(1)} - L_{e_1}^{(1)}\right), \tag{3.10}$$

and its p-value is denoted by $p_j^{(1)}$. Let the covariate corresponding to the smallest p-value be denoted by x_{e_2}, where the smallest p-value is determined by

$$p_{e_2}^{(1)} = \min\left(p_1^{(1)}, p_2^{(1)}, \ldots, p_{k-1}^{(1)}\right). \tag{3.11}$$

If $p_{e_2}^{(1)} < p_E$, then add x_{e_2} to the model and go to Step 2. Otherwise, end the procedure.

<u>Step 2</u>: This step includes a provision for backward elimination. The incorporation of a backward elimination check within what would normally be classified as a forward selection method gives the stepwise logistic regression procedure its name. For this step, the backward elimination check examines the possibility that once x_{e_2} is added to the model, x_{e_1} may no longer be significant. First, estimate a model containing both x_{e_1} and x_{e_2}, and let $L_{e_1,e_2}^{(2)}$ denote the log-likelihood of this model. Let $L_{-e_j}^{(2)}$ denote the log-likelihood of a model that does not contain x_{e_j}, where $j = 1, 2$. The likelihood ratio test statistic is now expressed as

$$G_{-e_j}^{(2)} = 2\left(L_{e_1,e_2}^{(2)} - L_{-e_j}^{(2)}\right) \tag{3.12}$$

Before deciding if a covariate should be removed from the model, a p-value for removal is defined, denoted p_R. This p-value must be assigned such that $p_R > p_E$ so that the stepwise procedure does not admit and expel the same covariate in consecutive steps. Converse to the task of admitting a covariate, the decision to remove a covariate from the model is made by identifying the largest p-value computed from the results of equation (3.12). This p-value is computed as

$$p_{r_2}^{(2)} = \max\left(p_{-e_1}^{(2)}, p_{-e_2}^{(2)}\right), \tag{3.13}$$

and the covariate associated with $p_{r_2}^{(2)}$ is denoted by x_{r_2}. If $p_{r_2}^{(2)} > p_R$, then x_{r_2} is removed from the model. Otherwise, x_{r_2} remains in the model, and Step 2 continues with the

forward selection phase. Now, estimate $k-2$ models, each containing x_{e_1}, x_{e_2}, and x_j, where $j=1,2,\ldots,k$ and $j \neq e_1, e_2$. Compute the log-likelihood for each of the $k-2$ models, and let x_{e_3} denote the covariate associated with the smallest p-value, where

$$p_{e_3}^{(2)} = \min\left(p_1^{(2)}, p_2^{(2)}, \ldots, p_{k-2}^{(2)}\right). \tag{3.14}$$

If $p_{e_3}^{(2)} < p_E$, then x_{e_3} add to the model and go to Step 3. Otherwise, end the procedure.

Step 3: The computations, model entry checks, and model removal checks are virtually the same as those of Step 2. The full model is estimated, using all of the covariates entered from previous steps. Reduced models are then fit by deleting each of the covariates from the full model, one at a time, with replacement. For example, if the k^{th} reduced model is estimated by deleting the i^{th} covariate from the full model, then the $(k+1)^{th}$ reduced model is estimated by deleting the $(i+1)^{th}$, or $(i-1)^{th}$, covariate from the full model, but including the i^{th} covariate. Log-likelihood values are computed for the full and reduced models, and likelihood ratio statistics comparing the full model to each of the reduced model are computed. The p-values corresponding to the likelihood ratio statistics are examined for both the backward elimination and forward selection phases. If the maximum p-value is greater than p_R, then the covariate corresponding to the maximum p-value is expelled from the model. Otherwise, the covariate corresponding to the maximum p-value is retained. If the minimum p-value is smaller than p_E, then the covariate corresponding to the minimum p-value is added to the model. Otherwise, the stepwise procedure ends.

Step 3 is repeated until one of two situations exist: either all k covariates have been added to the model, or all covariates in the model have p-values which are smaller

than p_R. In the latter situation, it must also be the case that all covariates not included in the model have p-values greater than p_E.

Summary

This chapter described the methodology used in this study. In addition to the logistic regression techniques presented in the previous chapter, the methods of data and variable manipulation were presented in detail in this chapter. All assumptions made about the data, as well as scaling and covariate selection techniques, were also presented. Chapter IV presents the results from the analyses conducted.

IV. Results and Analysis

Stepwise Regression

Because it contained the smallest sample size out of the three sets examined, the extra-state wars set was analyzed first. The analysis began first by dividing the data between 19th and 20th Century observations. Then, a model constructed from all 59 observations was obtained. Stepwise regression was performed on all cases for two purposes. One, the stepwise procedure fulfilled its customary role of identifying those covariates necessary to build an adequate logistic regression model on the response. Two, stepwise regression provided an adequate test for the rule of 10 described in the previous chapter. The results from stepwise regression are presented first.

Extra-State Wars (19th Century).

Hosmer and Lemeshow state that results from previous research on stepwise regression significance levels indicate that selecting p_E and p_R from the closed interval $[0.15, 0.20]$ yields the best results (Hosmer and Lemeshow, 2000:118). In addition, Hosmer's and Lemeshow's selections of p_E and p_R for an example experiment heavily influenced the entry and removal p-values selected for this research (Hosmer and Lemeshow, 2000:121). Using the values of $p_E = 0.15$ and $p_R = 0.2$ in MINITAB, the output of the analysis is shown in Figure 1

```
Stepwise Regression: Winner_ES_UN versus Dur_ES_UNS_1, C_Dths/Pop_E, ...

Alpha-to-Enter: 0.15  Alpha-to-Remove: 0.2
Response is Winner_ES_UNS_19 on 4 predictors, with N = 35
No variables entered or removed
```

Figure 1: Stepwise Results for 19th Century Extra-Systemic Wars

The fact that none of the covariates entered the model implied that each of the four p-values, corresponding to the likelihood ratio test statistic, was larger than p_E. The quantity for p_E could have been iteratively increased until at least one covariate entered the model. However, increasing p_E would have inflated the risk of allowing insignificant covariates to enter the model. This risk was already present, given that p_E was already larger than the overall significance level of $\alpha = 0.05$, but $p_E = 0.15$ was large enough such that a likelihood ratio test for an initial model would be significant at the 0.05 level. This was confirmed by fitting four univariate models in MINITAB and obtaining p-values for each likelihood ratio test. The MINITAB outputs for the four models are given in Figure 2 through Figure 5. The last p-value given for each model was the value in question.

Binary Logistic Regression: Winner_ES_UNS_19 versus Dur_ES_UNS_19

```
Variable          Value  Count
Winner_ES_UNS_19  1         27  (Event)
                  2          8
                  Total     35

Logistic Regression Table
                                                      Odds     95% CI
Predictor          Coef   SE Coef      Z      P  Ratio  Lower  Upper
Constant        1.25577  0.437887  2.87  0.004
Dur_ES_UNS_19  0.144370  0.583536  0.25  0.805   1.16   0.37   3.63

Log-Likelihood = -18.782
Test that all slopes are zero: G = 0.064, DF = 1, P-Value = 0.800
```

Figure 2: Univariate Logit Model (War Duration is Covariate)

The sample model with duration as its covariate had a 0.8 likelihood ratio p-value. Because $0.8 > 0.05$, the null hypothesis that all model coefficients are zero was not rejected. Thus, duration was not sufficient to explain the winner of a 19th Century extra-systemic war.

Binary Logistic Regression: Winner_ES_UNS_19 versus C_Dths/Pop_ES_UNS_19

```
Variable          Value  Count
Winner_ES_UNS_19  1         27  (Event)
                  2          8
                  Total     35

Logistic Regression Table
                                                           Odds     95% CI
Predictor                  Coef   SE Coef      Z      P  Ratio  Lower  Upper
Constant                1.21918  0.403617   3.02  0.003
C_Dths/Pop_ES_UNS_19  -0.132086  0.408161  -0.32  0.746   0.88   0.39   1.95

Log-Likelihood = -18.764
Test that all slopes are zero: G = 0.099, DF = 1, P-Value = 0.753
```

Figure 3: Univariate Logit Model (Proportion of State Population Killed is Covariate)

The sample model with the number of state combat deaths as a proportion of its population as the covariate had a 0.753 likelihood ratio p-value. Because $0.753 > 0.05$, the null hypothesis that all model coefficients are zero was not rejected. Thus, the number of state combat deaths as a proportion of its population was not sufficient to explain the winner of a 19th Century extra-systemic war.

Binary Logistic Regression: Winner_ES_UNS_19 versus C_Dths/Arm_ES_UNS_19

```
Variable          Value  Count
Winner_ES_UNS_19  1         27  (Event)
                  2          8
                  Total     35

Logistic Regression Table
                                                           Odds     95% CI
Predictor                  Coef   SE Coef       Z      P  Ratio  Lower  Upper
Constant                1.21314  0.403845    3.00  0.003
C_Dths/Arm_ES_UNS_19  -0.0552200  0.602345  -0.09  0.927   0.95   0.29   3.08

Log-Likelihood = -18.810
Test that all slopes are zero: G = 0.008, DF = 1, P-Value = 0.928
```

Figure 4: Univariate Logit Model (Proportion of State's Military Killed is Covariate)

The sample model with the number of state combat deaths as a proportion of its armed force size as the covariate had a 0.928 likelihood ratio p-value. Because $0.928 > 0.05$, the null hypothesis that all model coefficients are zero was not rejected. Thus, the number of state combat deaths as a proportion of its armed force size was not sufficient to explain the winner of a 19th Century extra-systemic war.

```
Binary Logistic Regression: Winner_ES_UNS_19 versus C_Dths/TDths_ES_

Variable          Value  Count
Winner_ES_UNS_19  1         27  (Event)
                  2          8
                  Total     35

Logistic Regression Table
                                                           Odds     95% CI
Predictor                   Coef   SE Coef     Z      P  Ratio  Lower  Upper
Constant                 1.24049  0.434328  2.86  0.004
C_Dths/TDths_ES_UNS_19  0.559073   1.43888  0.39  0.698   1.75   0.10  29.35

Log-Likelihood = -18.537
Test that all slopes are zero: G = 0.555, DF = 1, P-Value = 0.456
```

Figure 5: Univariate Logit Model (State's Proportion of Total Deaths is Covariate)

The sample model with the proportion of total combat deaths sustained by the participant as the covariate had a 0.456 likelihood ratio p-value. Because $0.456 > 0.05$, the null hypothesis that all model coefficients are zero was not rejected. Thus, the proportion of total combat deaths sustained by the participant was not sufficient to explain the winner of a 19th Century extra-systemic war.

The results clearly showed that each of the p-values was larger than $p_E = 0.15$, so the results from the stepwise procedure were confirmed. The focus then shifted to implications from the rule of 10. Out of $n = 35$ observations, $n_1 = 27$, $n_2 = 8$, and $m = \min(n_1, n_2) = 8$. Therefore, the resulting model should contain $k + 1 \le (m/10) = 0.8 \approx 0$ parameters. The rule of 10 proved effective in this case. As such, similar results were expected for the 20th Century observations.

While the p-value significance levels were chosen along the closed interval $[0.15, 0.2]$, the significance of model coefficients was determined by the p-values

corresponding to their individual Wald statistics. Each Wald statistic, as computed from equation (2.32), is given under the "Z" column in the MINITAB outputs. The values under the "P" column are the p-values for each Wald statistic, which should be smaller than $\alpha = 0.05$ in order to imply significance. Because each of these p-values for each of the covariates in the univariate models in Figures 4.1.1-2 – 4.1.1-5 was larger than 0.05, the implication was that not even an adequate univariate model could be fit using any of the available covariates for 19th Century extra-state wars. That is, a statistically significant relationship could not be established between the winner of a 19th Century extra-systemic war and the covariates selected from the COWP data. While discouraging, these results gave additional support to the results from the rule of 10.

Extra-State Wars (20th Century).

The results from the stepwise procedure in MINITAB for 20th Century extra-state wars are shown in Figure 6. The duration of the war and the proportion of the state's armed forces killed were selected by the stepwise process for entry into the model. The p-value for duration in Step 2 was very low, which implied that duration should prove significant in any main-effects models of the available covariates. The p-value for *C_Dths/Arm_ES_UNS_20*, however, was just barely smaller than the p-value for entry into the model. A model containing both of these covariates was estimated. It was expected that at least one of these covariates would be significant. The Wald statistics for this initial model were inspected to determine if one of the covariates should be eliminated. This model is discussed in a later section of this thesis.

```
Stepwise Regression: Winner_ES_UN versus Dur_ES_UNS_2, C_Dths/Pop_E, ...

Alpha-to-Enter:              0.15          Alpha-to-Remove:         0.2

Response is Winner_ES_UNS_20 on 4 predictors, with N = 24

Step                                1       2
Constant                        1.365   1.355

Dur_ES_UNS_20                   0.231   0.229
T-Value                          3.06    3.14
P-Value                         0.006   0.005

C_Dths/Arm_ES_UNS_20                    0.105
T-Value                                  1.66
P-Value                                 0.112
```

Figure 6: MINITAB Results for 20th Century Extra-State Wars

It was also interesting to note that while the rule of 10 proved valid for the 19th Century extra-state wars, it did not for the 20th Century data. From MINITAB, $n_1 = 13$, $n_2 = 11$, and $m = \min(13,11) = 11$. Therefore, the resulting model should contain no more than $k + 1 \le (m/10) = 1.1 \approx 1$ parameter. This implied that the model should contain $k = 0$ covariates; that is, a constant only model. It should be reiterated, however, that the rule of 10 is not absolute.

Extra-State Wars (Aggregated Data).

In addition to dividing up the observations between those of the 19th Century and those of the 20th Century, stepwise regression was also performed on extra-state wars using all $n = 59$ observations. The results are shown in Figure 7.

```
Stepwise Regression: Winner_ES_UN versus Duration_ES_, C_Deaths/Pop, ...

Alpha-to-Enter:           0.15     Alpha-to-Remove:            0.2


Response is Winner_ES_UNS on 4 predictors, with N = 59



Step                             1
Constant                      1.32

Duration_ES_UNS              0.161
T-Value                       2.75
P-Value                      0.008
```

Figure 7: Stepwise Results for Full Extra-State Wars Set

Here, the duration of the war was the only covariate significant enough to be included in the model at the settings used for this study. Furthermore, its p-value for the likelihood ratio test was again very small. It was expected that *Duration_ES_UNS* would demonstrate high significance in the estimated univariate model for predicting the winner of an extra-systemic war, which is discussed in a later section.

The rule of 10 provided a nearly accurate assessment in this case. Out of 59 observations, $m = \min(n_1, n_2) = \min(40,19) = 19$ was the minimum frequency, so the model should contain no more than $k + 1 \leq (19/10) = 1.9 \approx 1$ parameter. However, this result is so close to 2 that a univariate model, with duration as the independent variable, was believed to be adequate.

Intrastate Wars (19th Century).

The results from the stepwise regression procedure on 19th Century intrastate wars are given in Figure 8. As with the 19th Century extra-state wars, this implied that none of the four univariate models estimated possessed likelihood ratio p-values smaller than 0.15. Four univariate models, one for each of the covariates, were fit to show these large p-values. Figure 9 through Figure 12 give the MINITAB outputs for each of these four models, and the likelihood ratio p-values can be seen at the bottom of each figure. These figures showed that none of the covariates from the COWP data could be used to form a model predicting the winner of a 19th Century intrastate war.

Stepwise Regression: Winner_IS_UN versus Duration_IS_, Dead/Pop_IS_, ...

```
Alpha-to-Enter: 0.15  Alpha-to-Remove: 0.2
Response is Winner_IS_UNS_19 on 4 predictors, with N = 30
No variables entered or removed
```

Figure 8: Stepwise Results for 19th Century Intrastate Wars

Binary Logistic Regression: Winner_IS_UNS_19 versus Duration_IS_UNS_19

```
Variable                  Value    Count
Winner_IS_UNS_19              1       23  (Event)
                              2        7
                          Total       30

Logistic Regression Table
                                                         Odds       95% CI
Predictor                  Coef   SE Coef        Z      P Ratio      Lower    Upper
Constant                   1.32  0.602988     2.19  0.029
Duration_IS_UNS_19     0.249261  0.765408     0.33  0.745  1.28       0.29     5.75

Log-Likelihood =     -16.241
Test that all slopes are zero:    G =    0.114,    DF = 1,    P-Value =    0.735
```

Figure 9: Univariate Logit Model (Duration is Covariate)

The univariate model containing the duration of the conflict as the sole predictor of the winner possessed a 0.735 likelihood ratio p-value. Because $0.735 > 0.05$, the null hypothesis that all model coefficients are zero was not rejected. Thus, duration was not sufficient to explain the winner of a 19th Century intrastate war.

Binary Logistic Regression: Winner_IS_UNS_19 versus Dead/Pop_IS_UNS_19

Variable	Value	Count	
Winner_IS_UNS_19	1	23	(Event)
	2	7	
	Total	30	

Predictor	Coef	SE Coef	Z	P	Odds Ratio	95% CI Lower	Upper
Constant	1.98819	1.61659	1.23	0.219			
Dead/Pop_IS_UNS_19	3.63592	6.48808	0.56	0.575	37.94	0	12642267

Log-Likelihood = -15.819
Test that all slopes are zero: G = 0.958, DF = 1, P-Value = 0.328

Figure 10: Univariate Logit Model (Deaths per Population)

The univariate model containing the number of state combat deaths as a proportion of its population as the sole predictor of the winner possessed a 0.328 likelihood ratio p-value. Because $0.328 > 0.05$, the null hypothesis that all model coefficients are zero was not rejected. Thus, the proportion of state combat deaths to its population was not sufficient to explain the winner of a 19th Century intrastate war.

Binary Logistic Regression: Winner_IS_UNS_19 versus Dead/Arm_IS_UNS_19

Variable	Value	Count	
Winner_IS_UNS_19	1	23	(Event)
	2	7	
	Total	30	

Predictor	Coef	SE Coef	Z	P	Odds Ratio	95% CI Lower	95% CI Upper
Constant	2.16014	2.04746	1.06	0.291			
Dead/Arm_IS_UNS_19	3.09775	6.13261	0.51	0.613	22.15	0	3677249

Log-Likelihood = -16.029
Test that all slopes are zero: G = 0.539, DF = 1, P-Value = 0.463

Figure 11: Univariate Logit Model (Deaths per Total Armed Forces)

The univariate model containing the number of state combat deaths as a proportion of its armed force size as the sole predictor of the winner possessed a 0.463 likelihood ratio p-value. Because $0.463 > 0.05$, the null hypothesis that all model coefficients are zero was not rejected. Thus, the proportion of state combat deaths to its armed force size was not sufficient to explain the winner of a 19th Century intrastate war.

Binary Logistic Regression: Winner_IS_UNS_19 versus C_Dead/TotDead_IS_UNS_19

Variable	Value	Count	
Winner_IS_UNS_19	1	23	(Event)
	2	7	
	Total	30	

Predictor	Coef	SE Coef	Z	P	Odds Ratio	95% CI Lower	95% CI Upper
Constant	1.39879	0.508392	2.75	0.006			
C_Dead/TotDead_IS_UNS_19	-0.40011	0.398705	-1	0.316	0.67	0.31	1.46

Log-Likelihood = -15.786
Test that all slopes are zero: G = 1.025, DF = 1, P-Value = 0.31

Figure 12: Univariate Logit Model (Proportion of Total Casualties)

The univariate model containing the proportion of total combat deaths sustained by the participant as the sole predictor of the winner possessed a 0.31 likelihood ratio p-value. Because $0.31 > 0.05$, the null hypothesis that all model coefficients are zero was not rejected. Thus, the proportion of total combat deaths sustained by the participant was not sufficient to explain the winner of a 19th Century intrastate war.

As expected, each of the likelihood ratio p-values for each of the above models was larger than 0.15. The above figures also demonstrated that not even a statistically significant univariate model could be estimated for the 19th Century intrastate wars, as seen from the Wald statistic p-values being each much larger than the selected α significance level, 0.05.

In this case, the rule of 10 resisted scrutiny. That is, the results from the rule of 10 followed those obtained by stepwise regression. A model for the 19th Century intrastate wars should contain no more than $(m/10) = 0.7 \approx 0$ parameters, where $m = \min(n_1, n_2) = \min(23, 7) = 7$. Since each of the Wald statistic p-values was larger than 0.05, each of which failed to reject the null hypothesis of equation (2.29), no final model for the 19th Century intrastate war data was estimated. That is, a statistically significant relationship could not be established between the winner of a 19th Century intrastate war and the covariates derived from the COWP data.

Intrastate Wars (20th Century).

A total of $n = 43$ observations were available for analysis of 20th Century intrastate wars. The stepwise regression procedure suggested using three covariates in the model. The MINITAB output is shown in Figure 13. A binary logistic regression model, containing the three covariates suggested by the stepwise procedure, was estimated. The resulting parameter estimates, goodness-of-fit, and diagnostic measures are examined in a later section. The values for these measures influenced the substance of the final model.

Stepwise Regression: Winner_IS_UN versus Duration_IS_, Dead/Pop_IS_, ...

Alpha-to-Enter: 0.15 Alpha-to-Remove: 0.2

Response is Winner_IS_UNS_20 on 4 predictors, with N = 43

Step	1	2	3
Constant	1.317	1.322	1.334
Duration_IS_UNS_20	0.225	0.262	0.246
T-Value	3.56	3.94	3.77
P-Value	0.001	0	0.001
Dead/Arm_IS_UNS_20		-0.088	-0.214
T-Value		-1.56	-2.5
P-Value		0.127	0.017
Dead/Pop_IS_UNS_20			0.162
T-Value			1.92
P-Value			0.062

Figure 13: Stepwise Regression (20th Century Intrastate Wars)

It was also interesting to note that the stepwise results contradicted the rule of 10. Given that $n_1 = 26$ and $n_2 = 17$, the rule of 10 indicated that the model should only contain up to $(m/10) = (17/10) = 1.7 \approx 1$ parameter. If this result was accurate, then the stepwise regression should not have allowed any covariates to enter the model.

Intrastate Wars (Aggregated).

Stepwise regression was again conducted on aggregated data, this time for the $n = 73$ observations in the entire intrastate wars data set. It was interesting to confirm that the same single covariate, duration, was entered into the model for both this case and for the aggregated extra-state wars case. The results are shown in Figure 14. A possibility considered here was that a general relationship between the duration of both intrastate and extra-state wars and the winners of both may exist. The extent of such a relationship was examined after fitting the final models for both types of wars.

Stepwise Regression: Winner_IntS versus C_Dead/TotDe, Dead/Arm_...

```
Alpha-to-Enter:            0.15    Alpha-to-Remove:          0.2

Response is Winner_IntS on 4 predictors, with N = 73

Step                               1
Constant                       1.329

Duration_IntS_UNS              0.176
T-Value                         3.38
P-Value                        0.001
```

Figure 14: Stepwise Results (Aggregated Intrastate Wars)

In this case, the stepwise results correspond to those of the rule of 10. Given that $n_1 = 49$ and $n_2 = m = 24$, the rule of 10 suggests that the model can contain up to 2 parameters. Thus, it was expected that the univariate model using the duration of the conflict to predict the winner of an intrastate war would exhibit an adequate fit, and the length of the war would show statistical significance through its Wald p-value.

Interstate Wars (19th Century).

Using the values of $p_E = 0.15$ and $p_R = 0.2$, no covariates were entered into the model from the stepwise procedure, shown in Figure 15. As a test, the p-values for entry and removal were then incremented by 0.01 to determine just how large the entry p-value needed to be in order to admit even one covariate. It was found that the p-value for entry needed to be at least 0.31, and only the covariate *Dths/TDths_UNS_19* was admitted at $p_E = 0.31$, which is given in Figure 16. This result gave both additional support to the validity of the stepwise procedure and justification to the default level of p_E. In general, the p_E level necessary to include even one covariate in a multinomial model for predicting the outcome of an interstate war was too large to suggest that the resulting model correctly described the relationship between the outcome of an interstate war and the single predictor variable.

```
Stepwise Regression: Outcome(PR2) versus MilEx/TT_UNS,
Duration_UNS,...

Alpha-to-Enter:           0.15   Alpha-to-Remove:           0.2
Response is Outcome(PR2)_19 on 5 predictors, with N=58
No variables entered or removed.
```

Figure 15: Stepwise Results (Default Entry P-Value)

```
Stepwise Regression: Outcome(PR2) versus MilEx/TT_UNS,
Duration_UNS,...

Alpha-to-Enter:             0.31    Alpha-to-Remove:         0.35

Response is Outcome(PR2)_19 on 5 predictors, with N=58

Step                                1
Constant                        2.169

Dths/TDeaths_UNS_19               0.2
T-Value                          1.04
P-Value                         0.304
```

Figure 16: Stepwise Results (Incremented Entry P-Value)

It was found that the rule of 10 was again applicable in this case. The five outcome category frequencies were $n_1 = 25$, $n_2 = 16$, $n_3 = 2$, $n_4 = 10$, and $n_5 = 5$. Given that the smallest frequency was 2, the rule of 10 indicated that the model contain 0 parameters. That is, a correctly specified univariate multinomial model for predicting the outcome of a 19th Century interstate war could not be obtained using any of the covariates derived from the COWP data. The observations from the two outcomes with the lowest frequencies, 2 and 5, could have been eliminated and the stepwise procedure repeated. However, the rule of 10 would have then suggested at most a constant only model. The only way to have had the rule of 10 results reflect a model with at least 2 parameters; that is, a covariate and intercept, was to eliminate all 19th Century interstate war observations

except those corresponding to the first outcome. The problem would have then ceased to be a logistic regression problem, since a logistic regression problem requires a response variable with at least two categories. These limitations were only present in the COWP data. The COWP data on interstate wars contained too many missing entries, and the complete data were skewed in favor of victory by military imposition. Therefore, no further elimination of observations was performed, and no model for the 19th Century interstate wars was estimated.

Just as with binomial outcomes, the likelihood ratio test is also the basis of comparison in stepwise regression for multinomial outcomes. Univariate multinomial models were estimated, one for each of the five available covariates, and the MINITAB outputs for each model are shown in Figure 17 through Figure 21. The likelihood ratio statistic and its p-value is shown at the bottom of each figure. The purpose of estimating these models was to confirm the results from the stepwise procedure.

Nominal Logistic Regression: Outcome(PR2)_19 versus Dths/Pop_UNS_19

Response Information

Variable	Value	Count	
Outcome(PR2)_19	1	25	(Reference Event)
	5	5	
	4	10	
	3	2	
	2	16	
	Total	58	

Logistic Regression Table

					Odds	95% CI	
Predictor	Coef	SE Coef	Z	P	Ratio	Lower	Upper
Logit 1: (5/1)							
Constant	-1.54404	0.841252	-1.84	0.066			
Dths/Pop_UNS_19	0.292128	3.08618	0.09	0.925	1.34	0	567.39
Logit 2: (4/1)							
Constant	-1.32853	0.904292	-1.47	0.142			
Dths/Pop_UNS_19	-1.69837	3.28726	-0.52	0.605	0.18	0	114.97
Logit 3: (3/1)							
Constant	-8.30628	10.1307	-0.82	0.412			
Dths/Pop_UNS_19	-20.4914	34.2286	-0.6	0.549	0	0	1.72E+20
Logit 4: (2/1)							
Constant	-0.702933	0.679463	-1.03	0.301			
Dths/Pop_UNS_19	-1.07874	2.48216	-0.43	0.664	0.34	0	44.09

Log-Likelihood = -77.503
Test that all slopes are zero: G = 1.419, DF = 4, P-Value = 0.841

Figure 17: Univariate Multinomial Model (Deaths/Population)

The univariate multinomial model containing the proportion of participant combat deaths to its population exhibited a 0.841 likelihood ratio p-value. Because $0.841 > 0.05$, the null hypothesis that the coefficients in each logit are zero was not rejected. Thus, the proportion of participant combat deaths to its population was not sufficient to predict the outcome of a 19th Century interstate war.

Nominal Logistic Regression: Outcome(PR2)_19 versus Dths/Arm_UNS_19

Response Information

Variable	Value	Count	
Outcome(PR2)_19	1	25	(Reference Event)
	5	5	
	4	10	
	3	2	
	2	16	
	Total	58	

Logistic Regression Table

					Odds	95% CI	
Predictor	Coef	SE Coef	Z	P	Ratio	Lower	Upper
Logit 1: (5/1)							
Constant	-1.94081	1.12036	-1.73	0.083			
Dths/Arm_UNS_19	-2.11182	5.94128	-0.36	0.722	0.12	0	13809.2
Logit 2: (4/1)							
Constant	-1.51576	1.38604	-1.09	0.274			
Dths/Arm_UNS_19	-3.6547	7.63922	-0.48	0.632	0.03	0	82304.04
Logit 3: (3/1)							
Constant	-3.99637	4.71273	-0.85	0.396			
Dths/Arm_UNS_19	-8.52392	25.738	-0.33	0.741	0	0	1.61E+18
Logit 4: (2/1)							
Constant	-1.23185	1.43243	-0.86	0.39			
Dths/Arm_UNS_19	-4.70731	7.939	-0.59	0.553	0.01	0	51695.18

Log-Likelihood = -77.314
Test that all slopes are zero: G = 1.799, DF = 4,   P-Value = 0.773

Figure 18: Univariate Multinomial Model (Deaths/Armed Forces)

The univariate multinomial model containing the proportion of participant combat deaths to its armed force size exhibited a 0.773 likelihood ratio p-value. Because $0.773 > 0.05$, the null hypothesis that the coefficients in each logit are zero was not rejected. Thus, the proportion of participant combat deaths to its armed force size was not sufficient to predict the outcome of a 19th Century interstate war.

Nominal Logistic Regression: Outcome(PR2)_19 versus Dths/Tdeaths_UNS_19

Response Information

Variable	Value	Count	
Outcome(PR2)_19	1	25	(Reference Event)
	5	5	
	4	10	
	3	2	
	2	16	
	Total	58	

Logistic Regression Table

Predictor	Coef	SE Coef	Z	P	Odds Ratio	95% CI Lower	Upper
Logit 1: (5/1)							
Constant	-1.90113	0.596498	-3.19	0.001			
Dths/TDeaths_UNS_19	0.747108	0.51968	1.44	0.151	2.11	0.76	5.85
Logit 2: (4/1)							
Constant	-0.91742	0.375327	-2.44	0.015			
Dths/TDeaths_UNS_19	0.0167269	0.417589	0.04	0.968	1.02	0.45	2.31
Logit 3: (3/1)							
Constant	-2.80629	0.902243	-3.11	0.002			
Dths/TDeaths_UNS_19	0.731972	0.759506	0.96	0.335	2.08	0.47	9.21
Logit 4: (2/1)							
Constant	-0.471784	0.325654	-1.45	0.147			
Dths/TDeaths_UNS_19	0.185769	0.350366	0.53	0.596	1.2	0.61	2.39

Log-Likelihood = -76.755
Test that all slopes are zero: G = 2.917, DF = 4, P-Value = 0.572

Figure 19: Univariate Multinomial Model (Proportion of Total Dead)

The univariate multinomial model containing the proportion of total combat deaths sustained by the participant exhibited a 0.572 likelihood ratio p-value. Because $0.572 > 0.05$, the null hypothesis that the coefficients in each logit are zero was not rejected. Thus, the proportion of total combat deaths sustained by the participant was not sufficient to predict the outcome of a 19th Century interstate war.

Nominal Logistic Regression: Outcome(PR2)_19 versus Duration_UNS_19

Response Information

Variable	Value	Count	
Outcome(PR2)_19	1	25	(Reference Event)
	5	5	
	4	10	
	3	2	
	2	16	
	Total	58	

Logistic Regression Table

Predictor	Coef	SE Coef	Z	P	Odds Ratio	95% CI Lower	Upper
Logit 1: (5/1)							
Constant	-1.75906	0.595328	-2.95	0.003			
Duration_UNS_19	-0.488181	0.888294	-0.55	0.583	0.61	0.11	3.5
Logit 2: (4/1)							
Constant	-0.911427	0.385027	-2.37	0.018			
Duration_UNS_19	0.0259329	0.492012	0.05	0.958	1.03	0.39	2.69
Logit 3: (3/1)							
Constant	-3.02605	1.40983	-2.15	0.032			
Duration_UNS_19	-1.23568	2.32598	-0.53	0.595	0.29	0	27.75
Logit 4: (2/1)							
Constant	-0.520875	0.35039	-1.49	0.137			
Duration_UNS_19	-0.28124	0.485538	-0.58	0.562	0.75	0.29	1.96

Log-Likelihood = -77.644

Test that all slopes are zero: G = 1.139, DF = 4, P-Value = 0.888

Figure 20: Univariate Multinomial Model (War Duration)

The univariate multinomial model containing the duration of a 19th Century interstate war exhibited a 0.888 likelihood ratio p-value. Because $0.888 > 0.05$, the null hypothesis that the coefficients in each logit are zero was not rejected. Thus, the duration of a 19th Century interstate war was not sufficient to predict the outcome of a 19th Century interstate war.

Nominal Logistic Regression: Outcome(PR2)_19 versus MilEx/TT_UNS_19

Response Information

Variable	Value	Count	
Outcome(PR2)_19	1	25	(Reference Event)
	5	5	
	4	10	
	3	2	
	2	16	
	Total	58	

Logistic Regression Table

				Odds	95% CI	
Predictor	Coef	SE Coef	P	Ratio	Lower	Upper
Logit 1: (5/1)						
Constant	-25.8416	33.853	0.445			
MilEx/TT_UNS_19	-164.655	229.265	0.473	0	0	4.42E+123
Logit 2: (4/1)						
Constant	-20.2078	22.6403	0.372			
MilEx/TT_UNS_19	-131.193	153.546	0.393	0	0	5.30E+73
Logit 3: (3/1)						
Constant	5.79748	11.8392	0.624			
MilEx/TT_UNS_19	57.4596	82.236	0.485	9.00E+24	0	9.02E+94
Logit 4: (2/1)						
Constant	0.609505	8.56002	0.943			
MilEx/TT_UNS_19	7.23107	58.5934	0.902	1381.7	0	1.04E+53

Log-Likelihood = -76.989

Test that all slopes are zero: G = 2.447, DF = 4, P-Value = 0.654

Figure 21: Univariate Multinomial Model (Military Expenditures/Total Trade)

The univariate multinomial model containing the average amount of military spending as a proportion of the average total trade for a 19th Century interstate war exhibited a 0.654 likelihood ratio p-value. Because $0.654 > 0.05$, the null hypothesis that the coefficients in each logit are zero was not rejected. Thus, the average amount of military spending as a proportion of the average total trade for a 19th Century interstate war was not sufficient to predict the outcome of a 19th Century interstate war.

For Figure 21, the Z column was removed for two reasons. One, the values under the Z column were irrelevant to what was being demonstrated. That is, the likelihood ratio p-value for the model was the quantity of interest in each figure. Two, if the values under the Z column were needed, then they could be computed directly using equation (2.32), because the values labeled Z in MINITAB are equivalent to the Wald statistic, W.

It should be noted that the aforementioned stepwise selection results only applied to the available COWP data. Investigations of the outcomes of 19th Century interstate wars using other data sources may yield different stepwise selection results. It is imperative that the primary and secondary goals of this study be reiterated. The findings in this thesis appear only as a consequence of strictly using the COWP data. The purpose of subjecting the COWP data on interstate wars to stepwise selection was both to demonstrate the applicability of logistic regression to war termination studies and to expose the limitations of using open-source data.

Interstate Wars (20th Century).

The results from the rule of 10 for the $n = 167$ observations on 20th Century interstate wars did not match those from the stepwise selection, which admitted two covariates to the model. In this case, $n_1 = 62$, $n_2 = 37$, $n_3 = 29$, $n_4 = 16$, and $n_5 = 23$. With the fourth outcome having the smallest frequency, 16, the rule of 10 showed that the model should contain no more than 1 parameter. The results from stepwise selection in MINITAB, which are given in Figure 22, indicated otherwise.

Stepwise Regression: Outcome(PR2) versus MilEx/TT_UNS, Duration_UNS,...

```
Alpha-to-Enter:  0.15                                 Alpha-to-Remove:  0.2

Response is Outcome(PR2)_20 on 5 predictors, with N = 167

Step                                                   1              2
Constant                                            2.44          2.452

Dths/TDeaths_UNS_20                                 0.51           0.48
T-Value                                             4.99           4.67
P-Value                                                0              0

Duration_UNS_20                                                  -0.147
T-Value                                                           -1.51
P-Value                                                           0.133
```

Figure 22: Stepwise Selection (20th Century Interstate Wars)

Two covariates were selected for inclusion into the model: the duration of the war and the proportion of total deaths borne by the participant. Presented in a later section, this bivariate model was estimated, and the Wald statistics were examined to assess the individual significance of each covariate in the model. Once the final model was established, then the goodness-of-fit tests and diagnostic measures were analyzed.

The results in Figure 22 provided a starting point for constructing a multinomial prediction model for the outcome of 20^{th} Century interstate wars. The two covariates, duration and the proportion of total deaths borne by the participant, were used to estimate an initial model. It was expected that the goodness-of-fit statistics for the initial model would show it to be adequate. It was also expected that the likelihood ratio p-value for the initial model would support the notion that at least one of the included covariates was significant to predicting the outcome of a 20^{th} Century interstate war. The p-values for the Wald statistics suggested which covariate, if not both, was to be retained in the final model.

Interstate Wars (Aggregated).

Considering all $n = 225$ observations in the interstate wars data set, stepwise regression selected only one covariate: the proportion of total deaths borne by the participant, or *Deaths/TotDeaths_UNS*. Figure 23 shows the results from MINITAB. Additionally, computations from the rule of 10 supported the stepwise results. That is, the smallest outcome frequency was 26, so the rule of 10 concluded that the model could contain up to 2 parameters, making it at most a univariate multinomial model.

Stepwise Regression: Outcome(PR2) versus Deaths/Pop_U, Deaths/Arm_U,…

```
Alpha-to-Enter:  0.15                              Alpha-to-Remove:  0.2

Response is Outcome(PR2) on 5 predictors, with N = 225

Step                                                            1
Constant                                                    2.356

Deaths/TotDeaths_UNS                                        0.421
T-Value                                                      4.67
P-Value                                                         0
```

Figure 23: Stepwise Results (225 Interstate Wars)

Given that the covariate concerning casualty proportions was admitted in the stepwise results for both the aggregated interstate wars set and the 20th Century interstate wars set, the possibility that this covariate would be highly significant in both multinomial models was considered. The extent of this significance is discussed in later sections, where tests on individual model coefficients are conducted.

When the model for predicting the outcome of an interstate war was estimated, it was expected that the goodness-of-fit tests would show the model to be correctly specified. Additionally, the likelihood ratio test and Wald test was expected to indicate

the statistical significance of the casualty proportion covariate in predicting the outcomes of interstate wars.

Binary Logistic Regression Models on Winner

A total of four binary logistic regression models were estimated and analyzed. Instead of six models, as was postulated in the previous chapter, only four models were fit because in two out of the six possible cases, the stepwise regression procedure did not allow any covariates to enter the model. This result implied that in any univariate model for those cases, the resulting p-value for its likelihood ratio test statistic, *G*, was larger than the defined significance level for this research, $\alpha = 0.05$, the fact that it was also larger than $p_E = 0.15$ notwithstanding. Two binary models were estimated for the extra-state wars data: one for the 20th Century observations and the other for the aggregated data. The same was also done for the intrastate wars data. The results for the extra-state models are presented first.

<u>**20th Century Extra-State Wars Model.**</u>

The initial model estimated for the 20th Century extra-state wars data followed the recommendations from the stepwise selection procedure and contained two covariates: *Dur_ES_UNS_20* and *C_Dths/Arm_ES_UNS_20*. That is, stepwise regression considered the length of an extra-state war and the number of state deaths as a proportion of the state's military manpower as significant in predicting the winner of an extra-systemic war. The initial model fit from MINITAB is shown in Figure 24.

The initial model estimated for the 20th Century extra-state wars data contained those covariates identified by the stepwise procedure. Thus, the initial model was bivariate, containing the covariates concerning the duration of the conflict, *Dur_ES_UNS_20*, and the number of state deaths as a proportion of the state's military manpower, *C_Dths/Arm_ES_UNS_20*. Figure 24 shows the parameter estimates, Wald statistics, odds ratios, likelihood ratio test, deviance, Pearson chi-square test, and the Hosmer-Lemeshow test for the initial model.

Binary Logistic Regression: Winner_ES_ UN versus Dur_ES_U, C_Dths/Arm_E

```
Response Information

Variable                      Value    Count
Winner_ES_UNS_20                  1       13  (Event)
                                  2       11
                              Total       24

Logistic Regression Table
                                                                Odds   95% CI
Predictor                      Coef   SE Coef        Z      P Ratio Lower Upper
Constant                   0.478616  0.717177     0.67  0.505
Dur_ES_UNS_20              -1.08089  0.511024    -2.12  0.034   0.34  0.12  0.92
C_Dths/Arm_ES_UNS_20        -1.2966   2.11985    -0.61  0.541   0.27     0 17.43

Log-Likelihood = -11.31
Test that all slopes are zero:  G = 10.485,  DF = 2,  P-Value = 0.005

Goodness-of-Fit Tests

Method                   Chi-Square       DF        P
Pearson                     22.5125       21    0.371
Deviance                    22.6194       21    0.365
Hosmer-Lemeshow              6.0479        8    0.642
```

Figure 24: MINITAB Output (Initial Model)

The p-value 0.371 for the Pearson chi-square statistic was larger than α, which implied that the model using the duration of a 20th Century extra-systemic war and the proportion of state combat deaths to its armed force size to predict the winner of a 20th Century extra-state war was adequately fit. Similar implications were made from the p-values for the Deviance and Hosmer-Lemeshow statistics, which were $0.365 > 0.05$ and $0.642 > 0.05$, respectively.

Because the p-value for the likelihood ratio statistic was $0.005 < \alpha = 0.05$, the null hypothesis from equation (2.29) was rejected in favor of the alternative hypothesis, H_A. That is, there was sufficient evidence to suggest that at least one of the model coefficients was nonzero. The Wald statistics for each of the two covariates in the initial model were examined to determine which covariate, if not both, needed to be retained in the final model.

From the discussion of Wald statistics in Chapter 2 and equation (2.32), it follows that a Wald statistic with a p-value smaller than $\alpha = 0.05$ implies significance of the covariate under test. While the Wald p-value for *Dur_ES_UNS_20* was 0.034, the p-value for *C_Dths/Arm_ES_UNS_20* was 0.541, which suggested that the number of state deaths as a proportion of the state's military manpower was not significant to the model at an $\alpha = 0.05$ level. This result implied that a reduced model needed to be estimated. In spite of the initial model proving adequate, a reduced model was estimated that included only the *Dur_ES_UNS_20* covariate. A likelihood ratio test was then performed to compare the two models. The results of this comparison determined whether or not the reduced model was adequate enough to continue with odds ratio interpretation and diagnostics.

The logistic regression table along with the likelihood ratio test and goodness-of-fit tests for the reduced extra-state wars model is given in Figure 25.

Binary Logistic Regression: Winner_ES_UNS_20 versus Dur_ES_UNS_20

Response Information

Variable	Value	Count	
Winner_ES_UNS_20	1	13	(Event)
	2	11	
	Total	24	

Logistic Regression Table

					Odds	95% CI	
Predictor	Coef	SE Coef	Z	P	Ratio	Lower	Upper
Constant	0.568524	0.500277	1.14	0.256			
Dur_ES_UNS_20	-1.18758	0.516323	-2.3	0.021	0.3	0.11	0.84

Log-Likelihood = -12.575
Test that all slopes are zero: G = 7.953, DF = 1, P-Value = 0.005

Goodness-of-Fit Tests

Method	Chi-Square	DF	P
Pearson	25.3187	20	0.19
Deviance	25.1509	20	0.196
Hosmer-Lemeshow	11.8916	8	0.156

Figure 25: Reduced Model Results

As with the initial model, the p-value of the Wald statistic for *Dur_ES_UNS_20* was 0.021, which indicated that the duration of the conflict maintained its significance as a covariate. A likelihood ratio test was performed using the computation described in Section 0 to compare the reduced model to the initial model. This comparison was computed as two times the difference between the log-likelihood of the initial model and the log-likelihood of the reduced model. That is, $G = 2\left(-11.31 - \left(-12.575\right)\right) = 2.53$. The

critical chi-square value for this comparison was $\chi^2_{0.05,2} = 5.99$. Because the likelihood ratio statistic was smaller than the critical chi-square value, the reduced model, like the initial model, proved to be adequate. It was also noted that the likelihood ratio p-value for the reduced model in Figure 25 was $0.005 < 0.05$, which implied that at least one of the reduced model parameters was nonzero.

Since the three goodness-of-fit tests are statistically equivalent, the interpretation of the MINITAB output for each test was straight forward. MINITAB displayed the p-value for each statistic, which needed to be larger than $\alpha = 0.05$ to imply model adequacy. The deviance, Pearson chi-square, and Hosmer-Lemeshow statistics are each approximately distributed chi-square, and the computations differ only in their respective degrees of freedom. Each of the goodness-of-fit p-values was larger than α, which implied that the reduced model was also adequately fit. Thus, the final logit for the 20th Century extra-state data was expressed as

$$g\left(Dur\_ES\_UNS\_20\right) = 0.57 - \left(1.19 * Dur\_ES\_UNS\_20\right), \tag{4.1}$$

and the logistic regression model for predicting the probability of *Winner_ES_UNS_20* was given by

$$P\left(Winner \mid Duration\right) = \frac{e^{0.57-(1.19*Duration)}}{1+e^{0.57-(1.19*Duration)}}. \tag{4.2}$$

The covariate names for *Winner_ES_UNS_20* and *Dur_ES_UNS_20* were truncated to *Winner* and *Duration* for the purpose of explicitly stating the model in equation (4.2).

Odds Ratio Interpretation.

Since the reduced model was shown to be correctly specified, sufficient justification existed to continue with odds ratio interpretation. The odds ratio for the reduced model, at 0.3, implied an increased likelihood of the non-state actor emerging as the winner. Using the description surrounding equation (2.16) in Chapter 2, the odds ratio of 0.3 suggested that an extra-state war was 0.3 times as likely to end with the state as the victor than with the non-state participant as the winner, given a single-unit increase in the duration of the conflict. The 95% CI showed that this ratio could be as small as 0.11 or as large as 0.84. The tight range of the CI demonstrated a high level of confidence in the accuracy of the estimated odds ratio. The odds ratio was smaller than 1, so it actually implied that the non-state actor was more likely to win in a long war rather than the state. Defining a one-unit increase in war duration allowed a more accurate assessment of the odds ratio. Since unit normal scaling was used, the length of a single unit of war duration was denoted by the sample standard deviation of the extra-state wars duration data, which was computed to be 1426.19. By inverting the odds ratio, it followed that for about every 1426 days that an extra-state war lasts, the non-state participant is approximately 3.33 times as likely to emerge as the winner than the state participant. Therefore, in general, this result suggests that a long extra-systemic war favors the insurgency. This was a particularly unsettling finding, given that the United States has been engaged in the current war in Iraq for nearly 1460 days.

Diagnostics and Plots.

Three diagnostic plots were examined: ΔD_j versus $\hat{\pi}_j$, ΔX_j^2 versus $\hat{\pi}_j$, and $\Delta\hat{\beta}_j$ versus $\hat{\pi}_j$. Large values of ΔD_j and ΔX_j^2 indicated covariate patterns which were poorly fit. These values could be identified by being located in the top left or top right corners of the plots. Additionally, points far separated from the general pattern of the remaining points could also be classified as poorly fit (Hosmer and Lemeshow, 2000:176-179). Figure 26 is the plot of the change in model deviance versus the estimated probability of *Winner_ES_UNS_20*. The plot was examined for large values of ΔD_j. However, given that the goodness-of-fit tests showed the reduced model to be correctly specified, very few poorly fit covariate patterns were expected to appear.

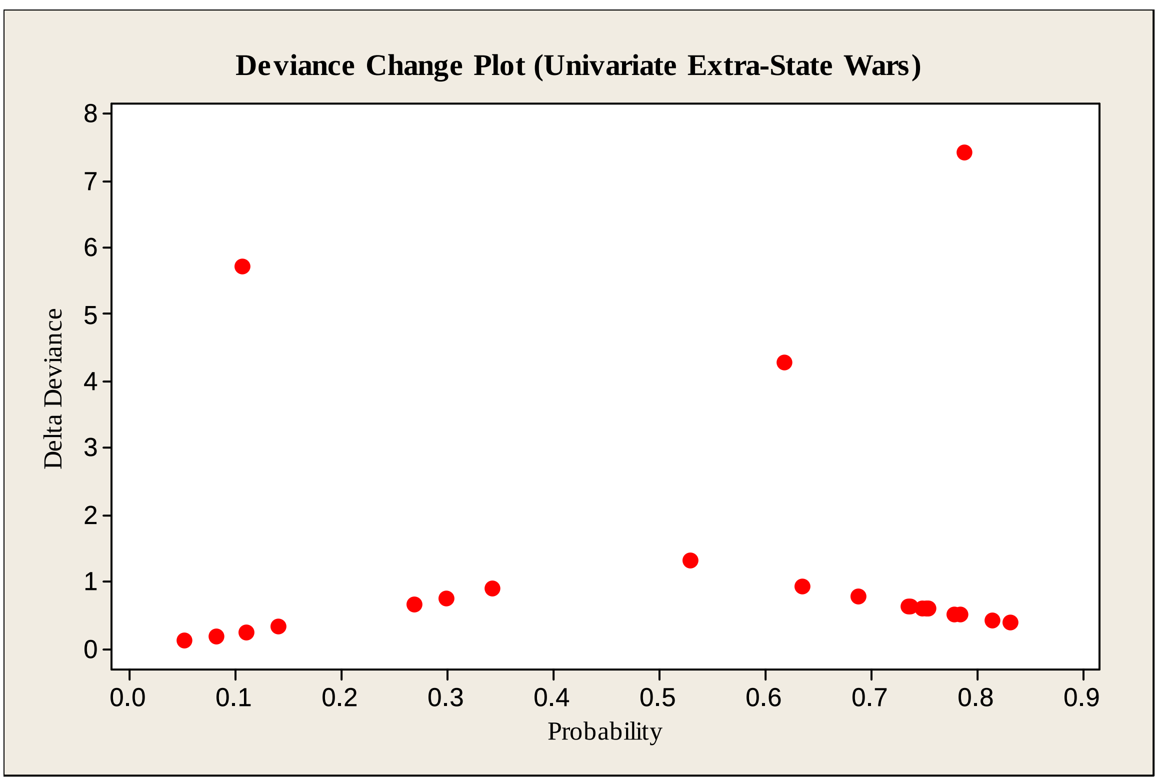

Figure 26: Deviance Change Plot for 20th Century Extra-State Wars Model

Three data points stood out as having large values for ΔD_j. This implied that the five observations corresponding to these three distinct values for *Dur_ES_UNS_20* were inadequately fit. The five observations were from the Italo-Libyan War of 1920, the Indonesian War of 1945, and the Western Saharan War of 1975. Their respective durations were 4444, 340, and 1334 days. These distinct values for duration exerted leverage on the fit of the model. Only 3 covariate patterns out of 20 were identified as poorly fit. Therefore, it was unnecessary to remove the 5 observations corresponding to these 3 covariate patterns and estimate a new model. Nonetheless, the extra-systemic wars identified above are generally unfamiliar in historical studies, and it could be beneficial to devote future statistical investigations to them.

The plot for the change in the Pearson chi-square statistic versus the estimated probability, shown in Figure 27, was also examined for inadequately fit data points. This plot indicated the same inadequately fit observations for duration as did the plot for the change in deviance. The model was assessed to be correctly fit, so having only 5 poorly fit observations out of $n = 24$ was considered acceptable. That is, sufficient evidence did not exist to imply that the model needed to be estimated again with the five observations removed.

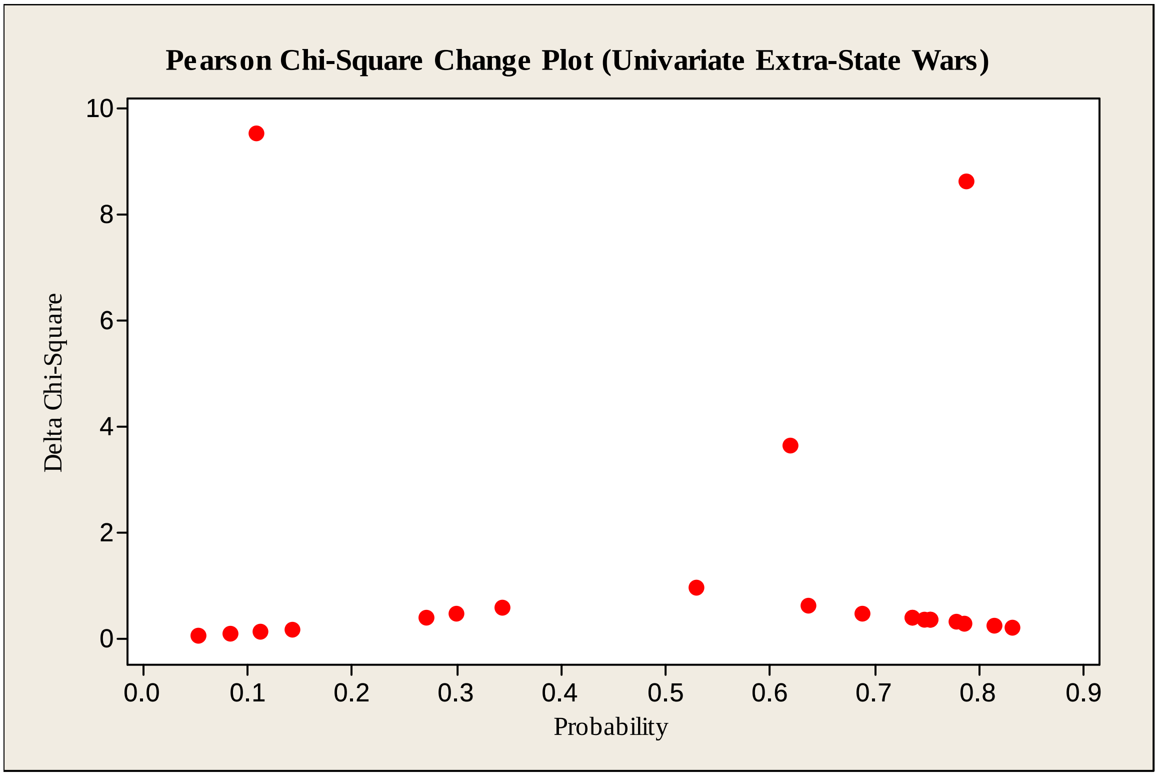

Figure 27: Pearson Statistic Change Plot for 20th Century Extra-State Wars Model

Large values of $\Delta\hat{\beta}_j$ were expected to exhibit similar characteristics within the plot in Figure 28 as both the large values of ΔD_j in Figure 26 and the large values of ΔX_j^2 in Figure 27. In contrast to the implications from large values of ΔD_j and ΔX_j^2, values of $\Delta\hat{\beta}_j$ that were both large and distanced from the general clustering of the remaining plotted points were flagged as influence points. Specifically, these flagged points corresponded to covariate patterns which had a significant effect on the values of the model parameters. Any influence diagnostic larger than 1 provided sufficient justification for deleting all observations corresponding to it and estimating a new model (Hosmer and Lemeshow, 2000:180).

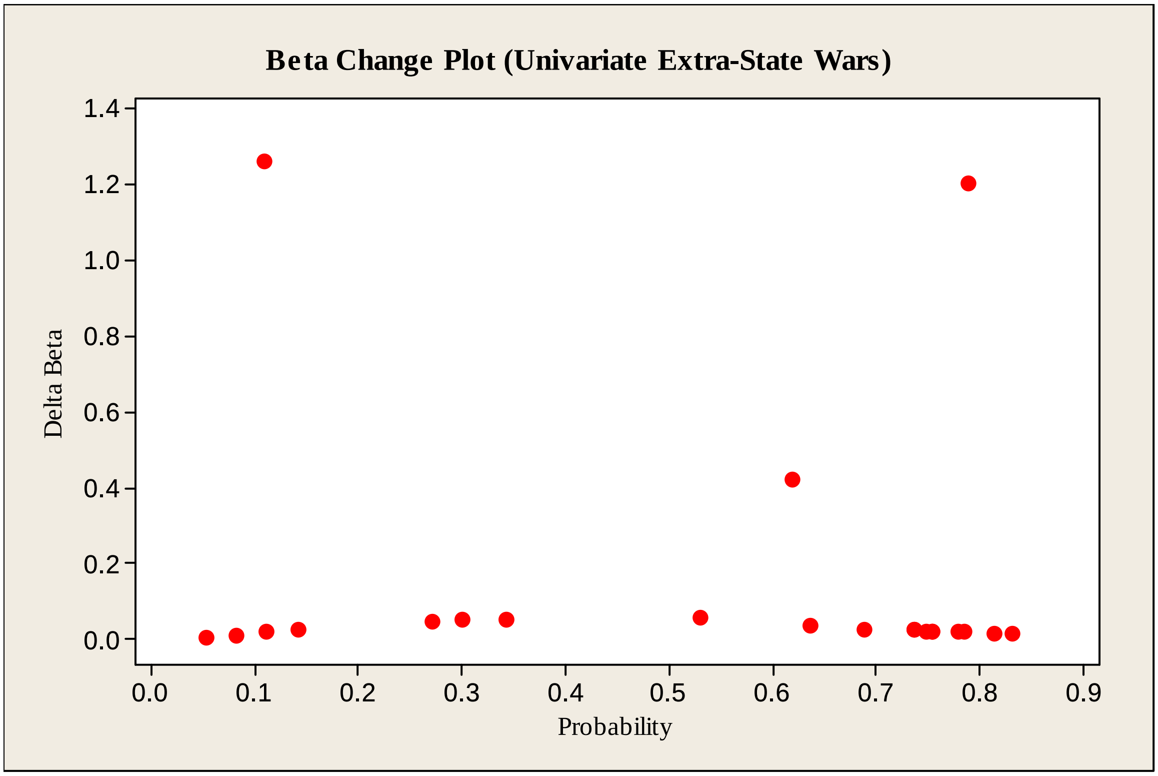

Figure 28: Coefficient Change Plot for 20th Century Extra-State Wars

There were two such covariate patterns in Figure 28 which were considered highly influential in parameter estimation. Three observations were identified by these covariate patterns: Italian participation in the Italo-Libyan War of 1920, British participation in the Indonesian War of 1945, and Dutch participation in the Indonesian War of 1945. Because of the high degree of influence these wars appeared to exert on the estimation of the model parameters, a future statistical investigation of these wars using a source with more complete and comprehensive data could be beneficial. Such an investigation could unveil the basis of the influence these wars had on the 20th Century model in this study. Considering that only two covariate patterns out of twenty were highly influential, the reduced model was deemed to be a generally good predictor of the winner in a 20th Century extra-state war.

The aforementioned observations were removed and a new model was estimated. Summary figures and diagnostic plots for this model are not given because only the changes in both the coefficients and odds ratios were of interest. The coefficient for duration in Figure 25 was $\hat{\beta}_1 = -1.19$. The coefficient in the revised model was computed to be $\hat{\beta}_1 = -4.96$, which was a notable change. The more drastic change, however, was in the odds ratio. The odds ratio for duration in Figure 25 was 0.3, but the odds ratio for duration in the revised model was 0.01. The odds ratio in the revised model showed that, with the influential observations deleted, the non-state faction was 100 times more likely to win a prolonged extra-systemic war in the 20th Century than was the state.

The drastic change in odds ratios from the reduced model to the revised model demonstrated the significant amount of influence that the two identified colonial wars had on a logistic regression model using duration to predict the winner of a 20th Century extra-systemic war. It is possible that other unidentified conditions existed within both the Italo-Libyan War and the Indonesian War that could account for their influence on the results of the model in this study. However, comprehensive data concerning these particular wars were not available from the COWP.

Aggregated Extra-State Wars Model.

The results from the stepwise procedure were use to justify estimating a univariate model with *Duration_ES_UNS* as the covariate. The logistic regression table is given in the MINITAB output of Figure 29.

Binary Logistic Regression: Winner_ES_UNS versus Duration_ES_UNS

```
Response Information

Variable                Value     Count
Winner_ES_UNS               1        40  (Event)
                            2        19
                        Total        59

Logistic Regression Table
                                                       Odds    95% CI
Predictor                Coef   SE Coef        Z     P Ratio Lower Upper
Constant             0.807665  0.298188     2.71 0.007
Duration_ES_UNS     -0.718876  0.295601    -2.43 0.015  0.49  0.27  0.87

Log-Likelihood = -33.768
Test that all slopes are zero: G = 6.614, DF = 1, P-Value = 0.010

Goodness-of-Fit Tests

Method             Chi-Square        DF        P
Pearson               55.4676        54    0.419
Deviance              64.7634        54     0.15
Hosmer-Lemeshow        5.1319         8    0.743
```

Figure 29: Logistic Regression Results for Aggregated Extra-State Wars

The p-value for the likelihood ratio test was 0.01, which suggested that at least one of the estimated parameters was nonzero, since $0.01 < 0.05$. The p-value for the Wald statistic on duration confirmed that *Duration_ES_UNS* was significant to the model, because $0.015 < 0.05$. Furthermore, the goodness-of-fit tests showed the model to be correctly specified, as each of the p-values was larger than $\alpha = 0.05$. The p-value for the deviance statistic, $D = 0.15$, was much smaller than that for both the Pearson chi-square and Hosmer-Lemeshow statistics. However, the degrees of freedom for both X^2 and D were identical, and the deviance statistic was larger than the Pearson chi-square statistic, so the smaller p-value for the deviance was understandable. The logit for this model is

$$g(Duration) = 0.807665 - (0.718876 * Duration) \tag{4.3}$$

The covariate label is truncated in equation (4.3) for the purpose of expressing the form of the logit. The covariate and response labels are truncated also to express the form of the logistic regression model for this case, which is

$$P(Winner \mid Duration) = \frac{e^{g(Duration)}}{1 + e^{g(Duration)}} \tag{4.4}$$

Equation (4.4) yields the conditional probability of the winner of an extra-systemic war, given that the war lasts a certain number of days. In general, the results in Figure 29 confirmed that the logistic regression model containing only the duration of the conflict was a good predictor of the winner of an extra-systemic war.

Odds Ratio Interpretation.

The odds ratio for the aggregated model was slightly larger than that for the 20th Century model. However, the odds ratio still favored the non-state actor. The state participant was 0.49 times as likely, or nearly half as likely, to win an extra-state war, for every approximate 1426-day increase in the duration of the war. Equivalently, the non-state participant was almost twice as likely to defeat the state force, for every 1426 days that the conflict continued. As with the 20th Century model, though to a lesser degree, it appeared that a long war strongly favored the non-state actor in an extra-systemic war.

Why was the non-state actor less likely to be victorious when the data were aggregated than when the 20th Century data were considered separately? One possible explanation involved considering the response frequencies between the two models. Specifically, for the 20th Century model, there were 11 observations in which the non-state actor won, while for the aggregated model, there were 19. Hence, the distribution of that response category between the two centuries was already skewed in favor of the 20th

Century. The addition of 8 observations where the non-state actor won in the aggregated model simply increased the likelihood of a non-state victory.

Another explanation could come from the specifics of the 19th Century extra-state wars, since most of these were colonial wars. Because the state participant was victorious in most of the 19th Century wars, additional statistical studies into the tactics and techniques used by these states may reveal the secrets to their successes.

Diagnostics and Plots.

The $\Delta\hat{\beta}_j$ plot, Figure 30, was examined first. It displayed a greater degree of separation between the influence points and the remaining observations. That is, the influence points were easier to identify in the $\Delta\hat{\beta}_j$ plot than the poorly fit covariate patterns were in either the ΔX_j^2 or ΔD_j plots.

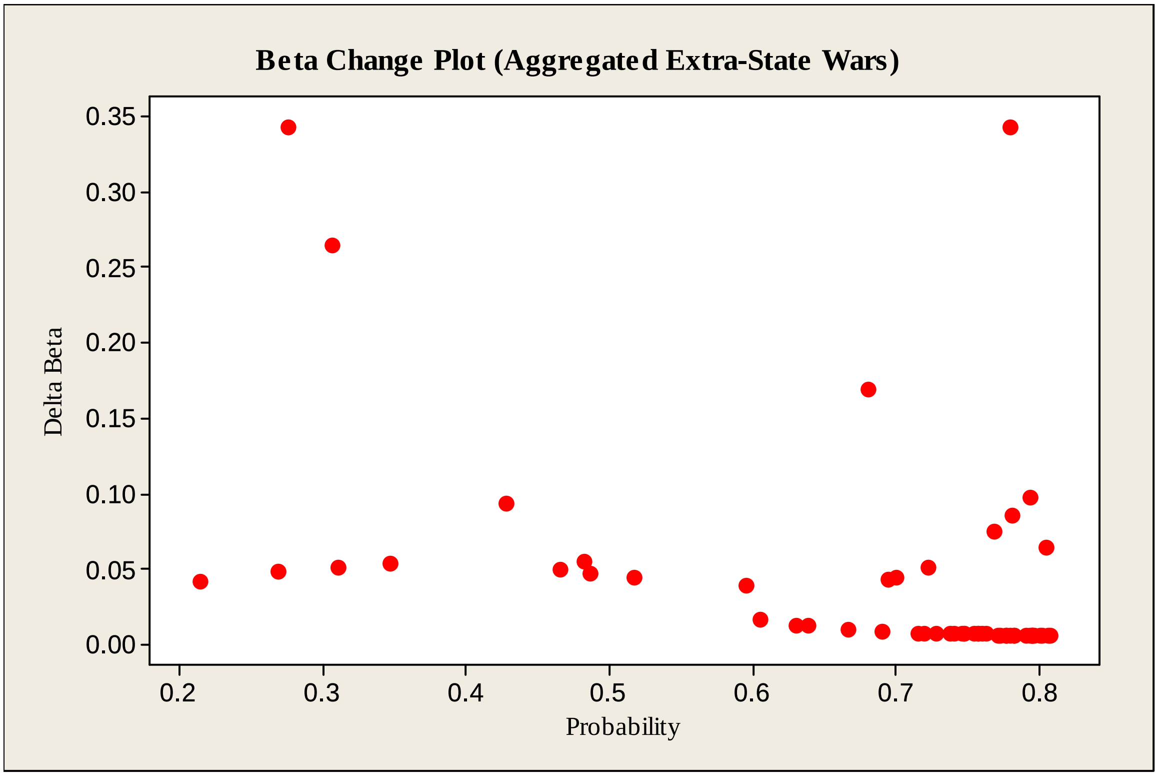

Figure 30: Beta Change Plot for Extra-State Wars (all n = 59 observations)

Four observations were designated as influence points. Their respective values for $\Delta\hat{\beta}_j$ were larger than those of the remaining data points. Three of these influence points corresponded to the same wars identified from the influence points for the 20th Century model. The fourth corresponded to the Franco-Tonkin War of 1873. These same influence points were identified in both the ΔX_j^2 and ΔD_j plots, which are given in Figure 31 and Figure 32.

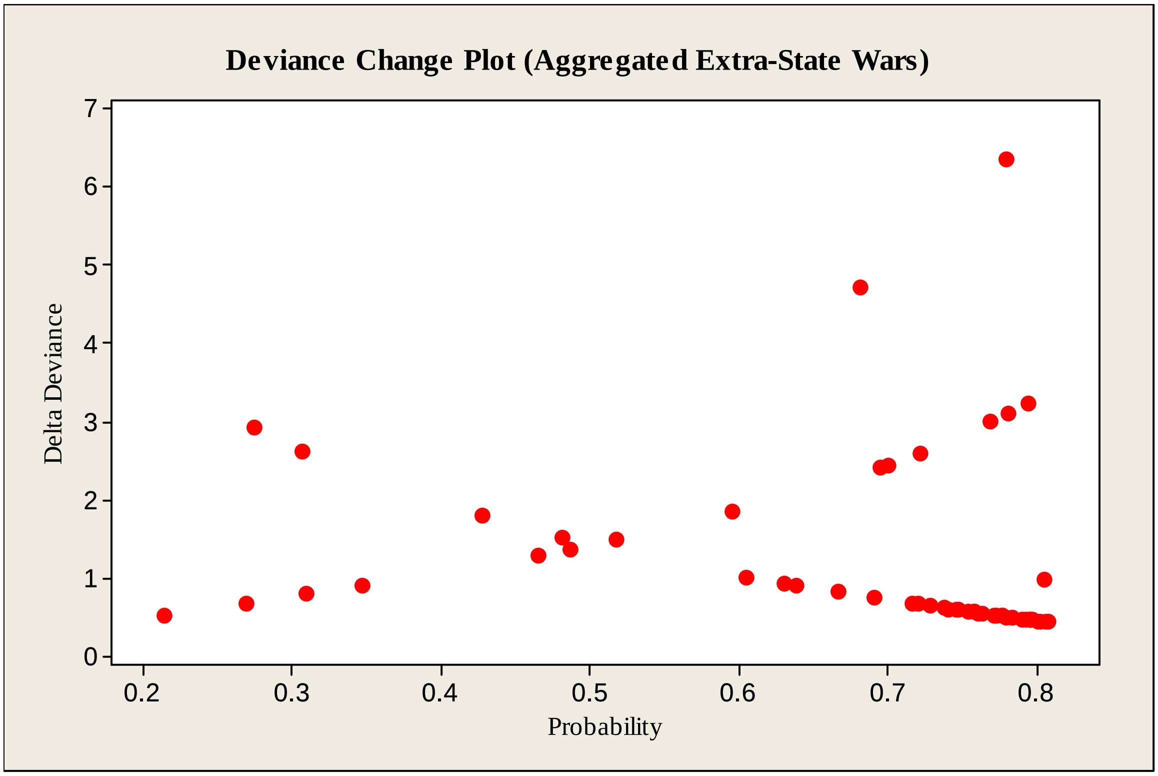

Figure 31: Deviance Change Plot for Extra-State Wars (all n = 59 observations)

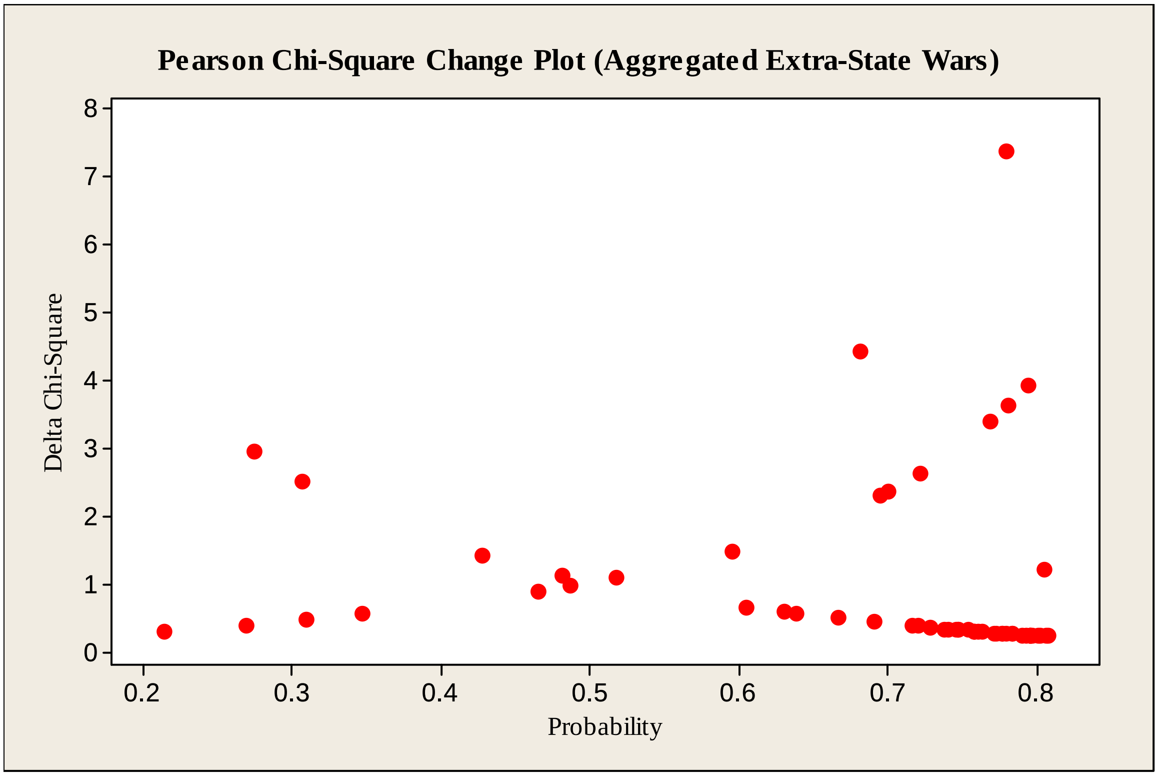

Figure 32: Chi-Square Change Plot for Extra-State Wars (n = 59 observations)

One course of action could have been to delete the influence points and estimate a new model. The four influence points only covered a range of $\Delta\hat{\beta}_j = 0.15$ to $\Delta\hat{\beta}_j = 0.35$. Based on the recommendation by Hosmer and Lemeshow that values of $\Delta\hat{\beta}_j > 1$ generally indicate the necessity for a new model, the influence points above were insufficiently large to justify fitting a new model (Hosmer and Lemeshow, 2000:180). Nonetheless, the six observations corresponding to the four influential covariate patterns were deleted, and a new model was estimated only to assess the change in odds ratios.

The odds ratio for this revised model, 0.27, revealed an even greater favor towards an insurgent victory than that of the original model. Rather than the non-state

faction being nearly two times more likely to win a long war, as was explained by the 0.49 odds ratio in the original model, the insurgency was now more than four times more likely to win a long war. Granted, the change in likelihood here was not as large as that from the 20th Century model, but the results were still disconcerting. It continually appears that some useful insights could be gained from additional investigations into the 19th Century extra-systemic wars, particularly those which were identified as influential in this study.

20th Century Intrastate Wars.

The initial model estimated for the 20th Century intrastate wars data followed the recommendations from the stepwise selection procedure and contained three covariates: *Duration_IS_UNS_20*, *C_Dead/TotDead_IS_UNS_20*, and *Dead/Pop_IS_UNS_20*. That is, stepwise regression considered the length of an intrastate war, the proportion of total deaths borne by the state participant, and the proportion of the total population of the state consumed by war deaths as significant in predicting the winner of an intrastate war. The initial model fit from MINITAB is shown in Figure 33.

Binary Logistic Regression: Winner_IS_UN versus Duration_IS_, Dead/Pop_IS_, ...

Response Information

Variable	Value	Count	
Winner_IS_UNS_20	1	26	(Event)
	2	17	
	Total	43	

Logistic Regression Table

Predictor	Coef	SE Coef	Z	P	Odds Ratio	95% CI Lower	Upper
Constant	1.26777	0.534039	2.37	0.018			
Duration_IS_UNS_20	-1.12622	0.411762	-2.74	0.006	0.32	0.14	0.73
Dead/Pop_IS_UNS_20	-0.359294	0.420409	-0.85	0.393	0.7	0.31	1.59
C_Dead/TotDead_IS_UNS_20	0.9499	0.699907	1.36	0.175	2.59	0.66	10.19

Log-Likelihood = -22.436
Test that all slopes are zero: G = 12.841, DF = 3, P-Value = 0.005

Goodness-of-Fit Tests

Method	Chi-Square	DF	P
Pearson	39.8724	39	0.431
Deviance	44.8716	39	0.239
Hosmer-Lemeshow	4.5651	8	0.803

Figure 33: Results for Initial Intrastate Wars Model (20th Century)

Each of the p-values for the goodness-of-fit tests were larger than 0.05, so those statistics showed the model to be adequately fit. Additionally, the p-value for the likelihood ratio test was 0.005, which was smaller than $\alpha = 0.05$. This result rejected the null hypothesis of equation (2.29) and indicated that at least one $\hat{\beta}_j$ was nonzero. The next task was to determine which of the three covariates were significant to the model. Thus, the p-value for each Wald statistic, from equation (2.32), was examined to determine covariate significance.

The p-value for each Wald statistic is found in the fourth column of the logistic regression table in Figure 33. Only one of the three covariates was found to be

significant. The p-values for *C_Dead/TotDead_IS_UNS_20* and *Dead/Pop_IS_UNS_20* were 0.175 and 0.393, respectively. Since $0.175 > 0.05$ and $0.393 > 0.05$, neither of these covariates were significant. The question was raised as to why these two covariates were allowed to enter the model in the stepwise selection procedure but were not truly significant to it. The purpose of stepwise selection was to provide guidance in constructing an adequate model. That is, the results from stepwise regression provided a set of covariates which would yield a logistic regression model deemed adequate by the goodness-of-fit tests. Consequently, individual significance was not part of the stepwise assessment. The p-value for *Duration_IS_UNS_20*, however, did imply significance, since $0.006 < 0.05$. The next logical step was to fit a new logistic regression model containing only *Duration_IS_UNS_20*.

The MINITAB output for the reduced model is given in Figure 34. The model was univariate and took the form of equation (2.1). The goodness-of-fit p-values were each again larger than $\alpha = 0.05$, which implied model adequacy. The p-value for the Wald statistic of *Duration _IS_UNS_20* was 0.004, which confirmed that *Duration _IS_UNS_20* maintained its position as a significant predictor of the winner of a 20th Century intrastate war. The results from the goodness-of-fit and Wald tests showed that not only was the reduced model adequate, but also that it was correctly specified from the available COWP data.

Binary Logistic Regression: Winner_IS_UNS_20 versus Duration_IS_UNS_20

```
Response Information

Variable                   Value    Count
Winner_IS_UNS_20               1       26 (Event)
                               2       17
                           Total       43

Logistic Regression Table
                                                       Odds   95% CI
Predictor                   Coef  SE Coef       Z     P Ratio Lower Upper
Constant                0.867621 0.390469    2.22 0.026
Duration_IS_UNS_20      -1.08163 0.375676   -2.88 0.004  0.34  0.16  0.71

Log-Likelihood = -23.504
Test that all slopes are zero: G = 10.706, DF = 1, P-Value = 0.001

Goodness-of-Fit Tests

Method               Chi-Square       DF        P
Pearson                 42.6244       39    0.318
Deviance                47.0073       39    0.177
Hosmer-Lemeshow          8.4269        8    0.393
```

Figure 34: Reduced Model Results

Odds Ratio Interpretation.

The odds ratio, at 0.34, suggested that the rebel or insurgent faction was more likely to win an intrastate war, given a single-step increase in the duration of the conflict. The reference category for *Winner_IS_UNS_20* was 1, corresponding to a state victory. Therefore, the odds ratio needed to be larger than 1 in order to imply a greater likelihood of the state winning an intrastate war than the non-state actor. Just as with the extra-state models, a unit-length increase in duration needed to be defined such that the odds ratio could be accurately interpreted. After reversing the unit normal scaling procedure described by equation (3.6), a one-step change in intrastate war duration was found to be

approximately 1679 days. Thus, given a duration of slightly more than four and a half years, the rebel faction was nearly three times more likely to emerge victorious than the state from an intrastate war in the 20th Century.

The combination of goodness-of-fit tests, diagnostic plot examination, and odds ratio interpretation demonstrated that for the variables and data available from the COWP, a univariate model containing the duration of an intrastate war adequately predicted the winner of the conflict. The logit for the reduced model was expressed as

$$g\left(Duration\right) = 0.86762 - \left(1.08163 * Duration\right), \tag{4.5}$$

and the binary logistic regression model for 20th Century intrastate wars was given by

$$P\left(Winner \mid Duration\right) = \frac{e^{g\left(Duration\right)}}{1 + e^{g\left(Duration\right)}}. \tag{4.6}$$

Again, the covariate labels *Duration_IS_UNS_20* and *Winner_IS_UNS_20* were truncated for the purpose of explicitly expressing the logit and binary model. Equation (4.6) yields the conditional probability of the winner of an intrastate war, given that the war lasts a certain number of days.

Diagnostics and Plots.

The diagnostic plots were examined next to locate influence points. The plots for ΔX_j^2, ΔD_j, and $\Delta\hat{\beta}_j$ are given in Figure 35, Figure 36, and Figure 37, respectively.

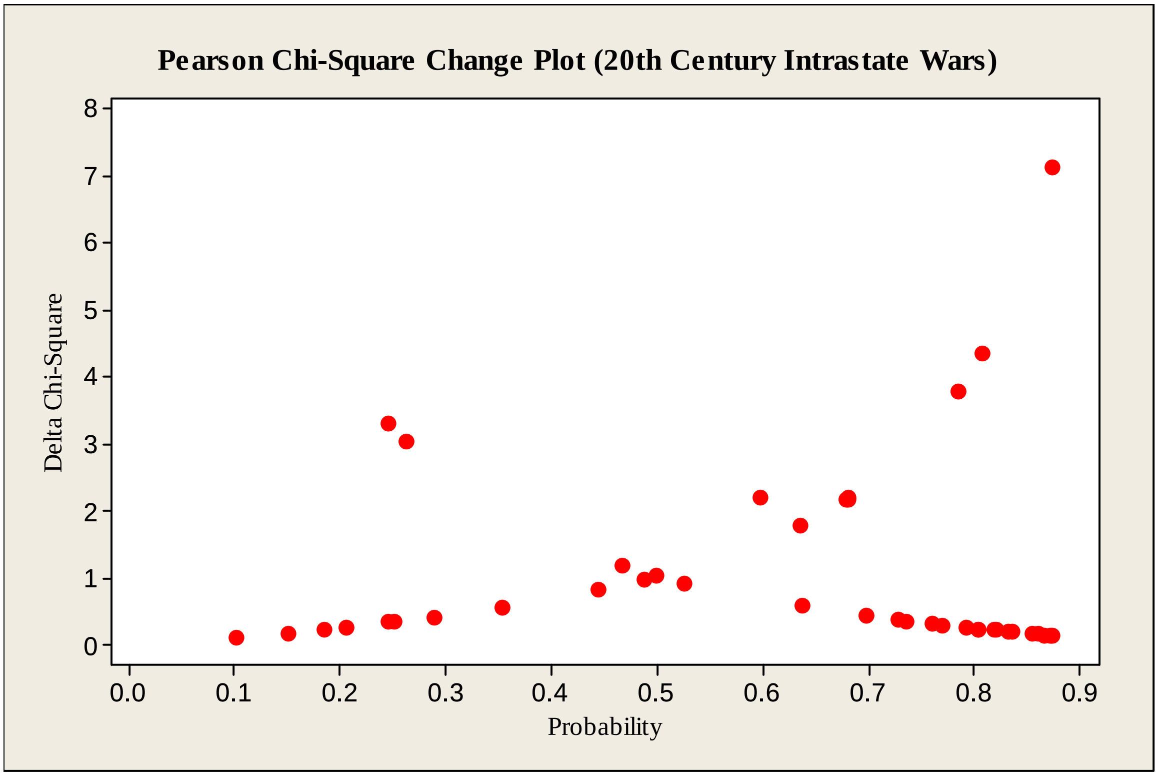

Figure 35: Chi-Square Change Plot for Reduced Intrastate Wars Model

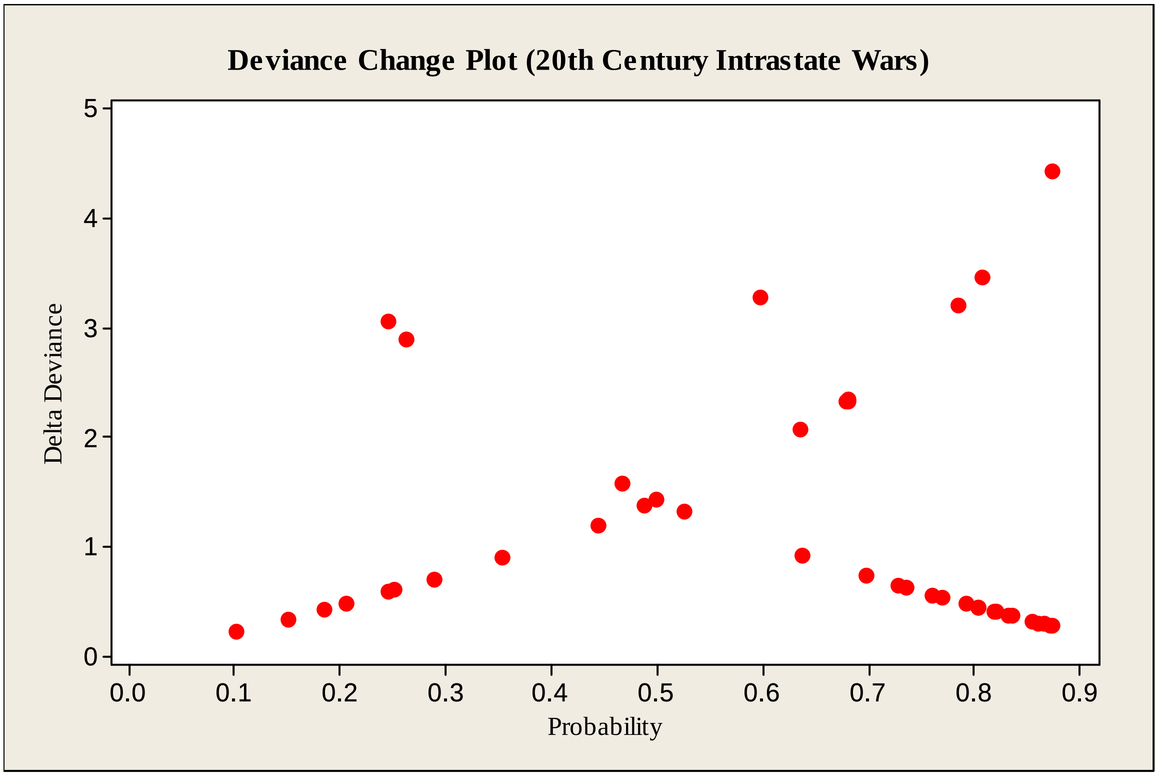

Figure 36: Deviance Change Plot for Reduced Intrastate Wars Model

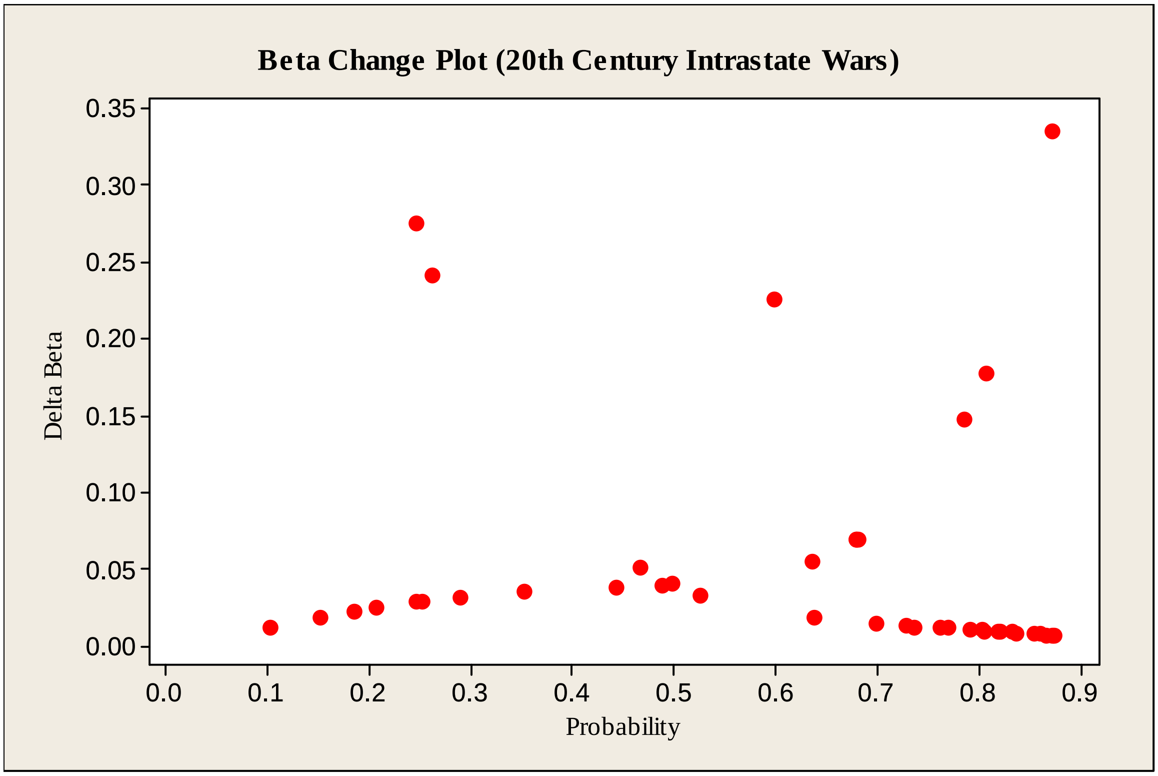

Figure 37: Beta Change Plot for Reduced Intrastate Wars Model

It was easy to identify six influence points from the $\Delta\hat{\beta}_j$ plot. The poorly fit points were not as apparent in either the ΔX_j^2 plot or the ΔD_j plot. The six influence points corresponded to five 20th Century intrastate wars: the Cambodia-Khmer Rouge War of 1970, the Pinochet Rebellion in Chile in 1973, the Somali Secession from Ethiopia in 1976, the Communist Rebellion in El Salvador in 1979, and the Renamo Rebellion in Mozambique in 1979. The data for this research were organized at the participant level, so the six influence points concerned specific actors in the aforementioned intrastate wars. Table 8 gives the war, participant, $\Delta\hat{\beta}_j$ value, and duration of involvement identified from the influence points.

Table 8: Observations Identified by Influence Points

Intrastate War	Participant	Beta Change	Duration (days)
Cambodia vs. Khmer Rouge	United States	0.147369	977
	Republic of Vietnam	0.178397	766
Chile vs. Pinochet Rebels	Chile	0.335365	5
Ethiopia vs. Somali Rebels	Somali Rebels	0.225726	2376
El Salvador vs. Salvadorean Democratic Front	El Salvador	0.242076	4599
Mozambique vs. Renamo	Mozambique	0.275127	4733

The largest of these $\Delta\hat{\beta}_j$ values was about 0.34, which is smaller than 1, so the magnitudes of the influence points were not sufficient to justify deleting the six observations and fitting a new model.

Aggregated Intrastate Wars.

The aggregated intrastate was a univariate model containing *Duration_IntS_UNS* as the independent variable, as recommended from the stepwise selection procedure. The results from the model estimation are given in Figure 38. The Pearson chi-square, Deviance, and Hosmer-Lemeshow goodness-of-fit tests showed the model to be adequate. Each of the p-values for the goodness-of-fit statistics was larger than $\alpha = 0.05$, as required for implying a good model fit. The p-value for the likelihood ratio test was 0.002, so the null hypothesis that all model coefficients are zero was rejected. Thus, the p-value for the Wald statistic on *Duration_IntS_UNS* was examined to determine the individual significance of the covariate. Since $0.003 < 0.05$, it was concluded that the duration of the conflict was significant to the model predicting the winner.

Binary Logistic Regression: Winner_IntS versus Duration_IntS_UNS

Response Information

Variable	Value	Count	
Winner_IntS	1	49	(Event)
	2	24	
	Total	73	

Logistic Regression Table

Predictor	Coef	SE Coef	Z	P	Odds Ratio	95% CI Lower	Upper
Constant	0.784946	0.270836	2.9	0.004			
Duration_IntS_UNS	-0.804099	0.271266	-2.96	0.003	0.45	0.26	0.76

Log-Likelihood = -41.264
Test that all slopes are zero: G = 9.934, DF = 1, P-Value = 0.002

Goodness-of-Fit Tests

Method	Chi-Square	DF	P
Pearson	73.2111	68	0.311
Deviance	82.5286	68	0.111
Hosmer-Lemeshow	4.4029	8	0.819

Figure 38: Results for Univariate Intrastate Wars Model

Odds Ratio Interpretation.

The odds ratio, at 0.45, suggested that the non-state actor was still more likely to win an intrastate war, given a single-step increase in the duration of the conflict. Just as with the 20^{th} Century intrastate model, a unit-length increase in duration was approximately 1679 days. Thus, given a duration of slightly more than four and a half years, the rebel faction was over two times more likely to emerge victorious than the state from an intrastate war.

The combination of goodness-of-fit tests, diagnostic plot examination, and odds ratio interpretation demonstrated that for the variables and data available from the

COWP, a univariate model containing the duration of an intrastate war adequately predicted the winner of the conflict. The logit for the reduced model was expressed as

$$g\left(Duration\right) = 0.78 - \left(0.8 * Duration\right), \tag{4.7}$$

and the binary logistic regression model for aggregated intrastate wars was given by

$$P\left(Winner \mid Duration\right) = \frac{e^{g(Duration)}}{1 + e^{g(Duration)}}. \tag{4.8}$$

Again, the covariate labels *Duration_IntS_UNS* and *Winner_IS_UNS* were truncated for the purpose of explicitly expressing the logit and binary model. Equation (4.8) yields the conditional probability of the winner of an intrastate war, given that the war lasts a certain number of days.

Diagnostics and Plots.

The diagnostic plot for $\Delta\hat{\beta}_j$ is given in Figure 39. Four influence points were clearly distinguished in the plot. The four influence points corresponded to the following intrastate wars: the Russo-Circasian War of 1829, the Somali Secession from Ethiopia in 1976, the Communist Rebellion in El Salvador in 1979, and the Renamo Rebellion in Mozambique in 1979. The observations corresponding to the influence points concerned the following participants: Russia, Somali rebels, El Salvador, and Mozambique. However, the $\Delta\hat{\beta}_j$ values for these influence points were much smaller than 1, so there was insufficient evidence to suggest deleting these observations and fitting a new model.

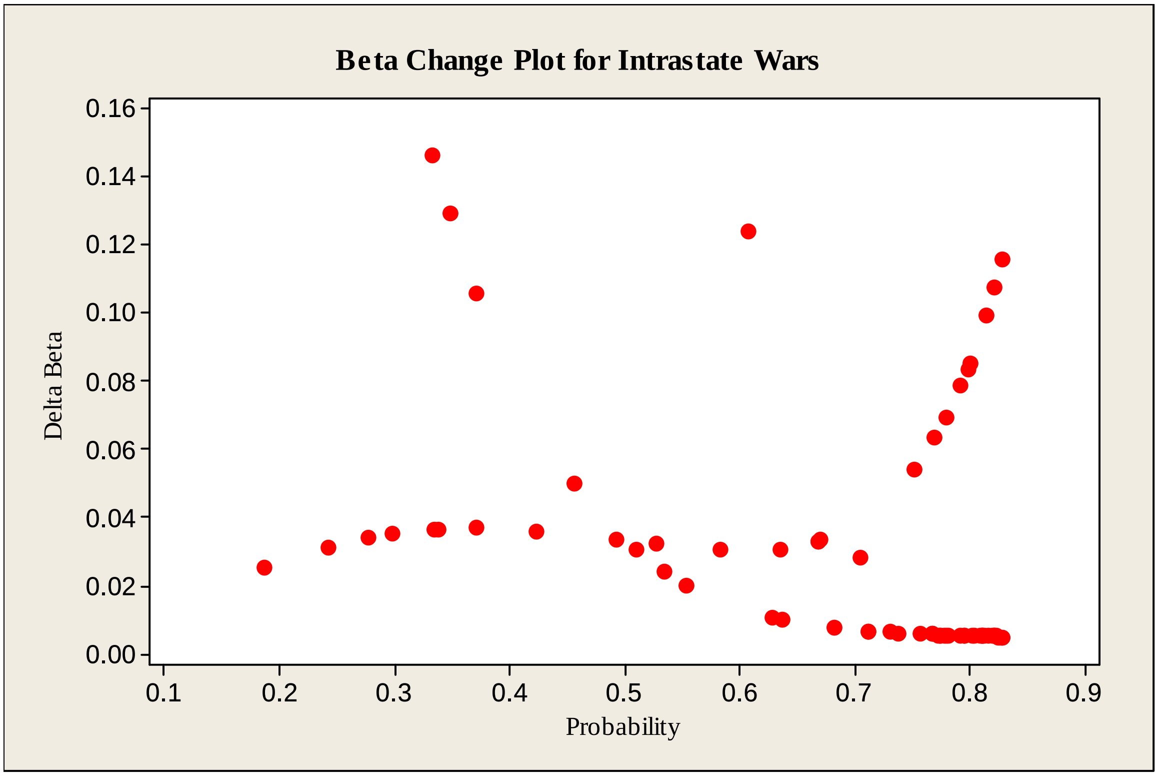

Figure 39: Beta Change Plot for Aggregated Intrastate Wars Model

The poorly fit covariate patterns were not as easily identified in either the ΔD_j plot or the ΔX_j^2 plot. Two patterns were identified as poorly fit. That is, 2 of the 68 distinct covariate values did not follow the general pattern of the plots as did the remaining 66. Four observations corresponded to these poorly fit covariate patterns. The plot for ΔD_j is shown in Figure 40, and the plot for ΔX_j^2 is shown in Figure 41.

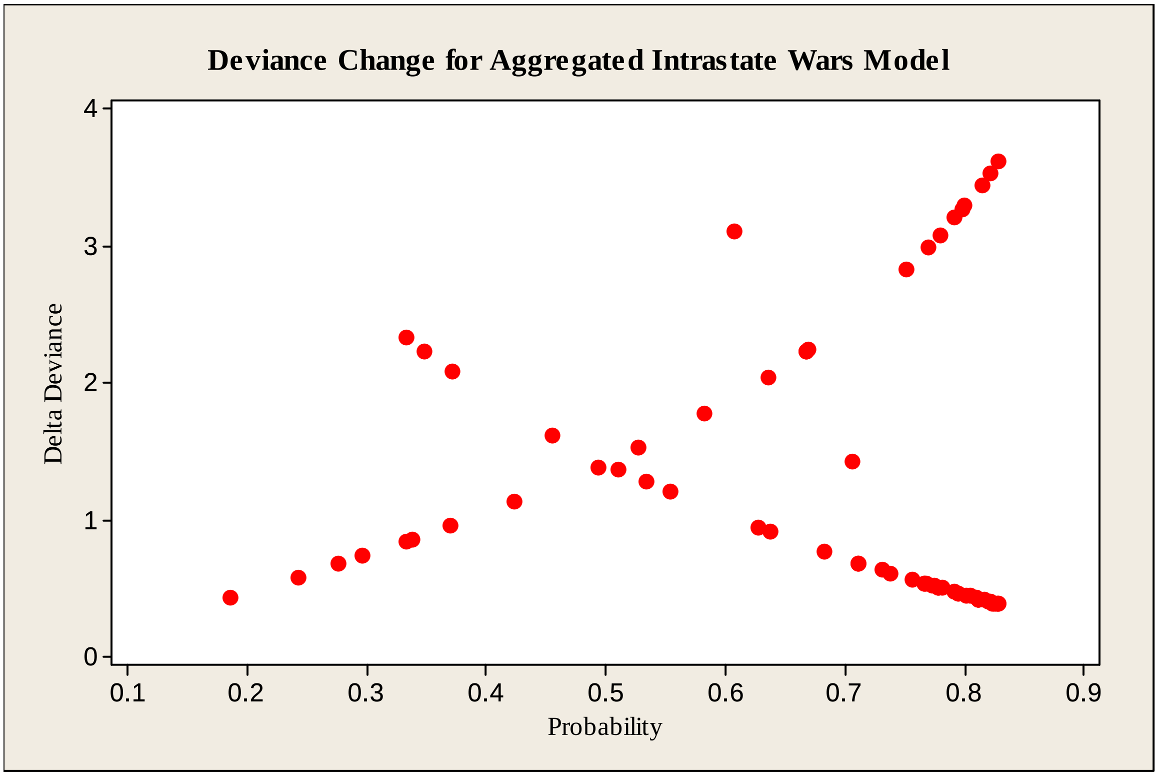

Figure 40: Deviance Change Plot for Aggregated Intrastate Wars

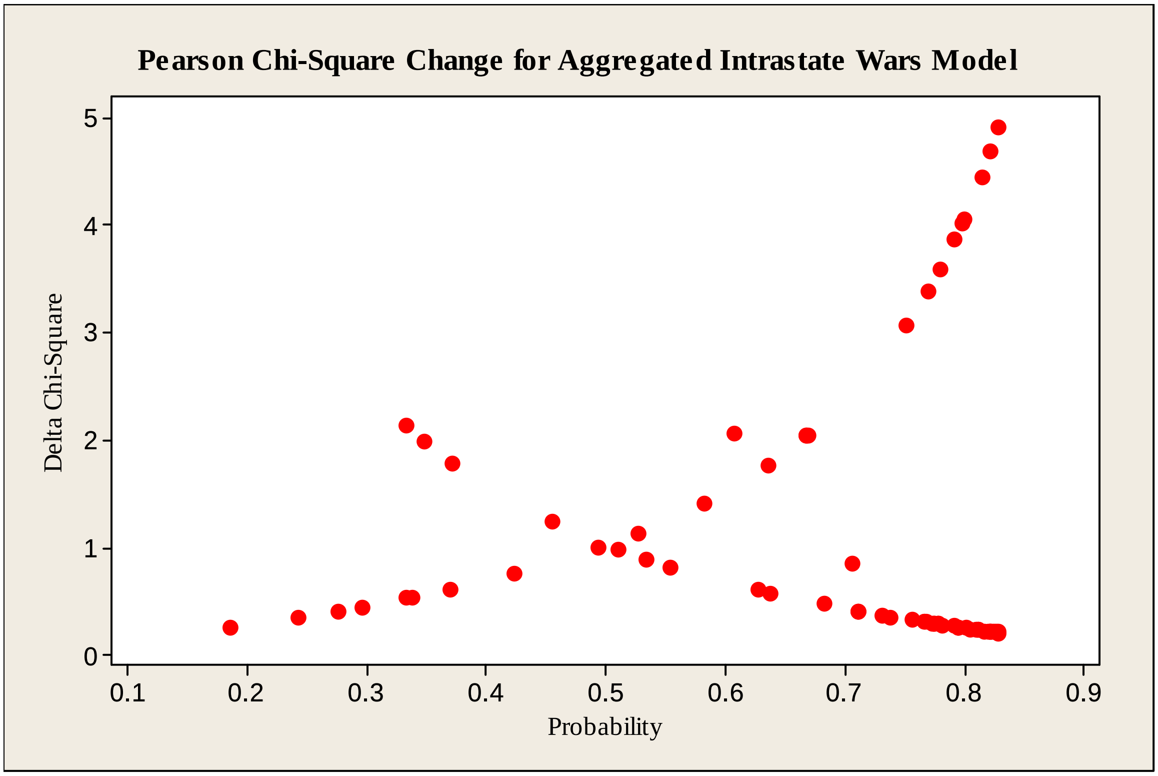

Figure 41: Pearson Chi-Square Change Plot for Intrastate Wars

The intrastate wars corresponding to the two poorly fit patterns were the War Between the States and the Somali Secession from Ethiopia in 1976. Since only 4 out of $n = 73$ observations were associated with these patterns, there was insufficient evidence to suggest deleting the 4 data points and estimating a new model. Furthermore, the ΔD_j and ΔX_j^2 values for these 2 patterns were moderate in relation to the rest of the points on the plots, so noting the range on which their estimated probabilities lied gave additional insights into the amount of leverage they exerted on the estimation of the model coefficients.

For Figure 41, the data point for Union involvement in the War Between the States possessed a delta chi-square value of $\Delta X_{23}^2 = 0.86$, delta deviance value of

$\Delta D_{23} = 1.43$, and a leverage value of $h_{23} = 0.03$. Its estimated probability falling within the region $0.7 < \hat{\pi}(x_{23}) < 0.9$ implied that its leverage was moderate, compared to the other observations (Hosmer and Lemeshow, 2000:175). An examination of the plots of ΔX_j^2 versus h_j and ΔD_j versus h_j, given in Figure 42 and Figure 43, respectively, showed this to be the case. That is, its leverage value was sufficiently large to have a moderate effect on the estimation of the model parameters.

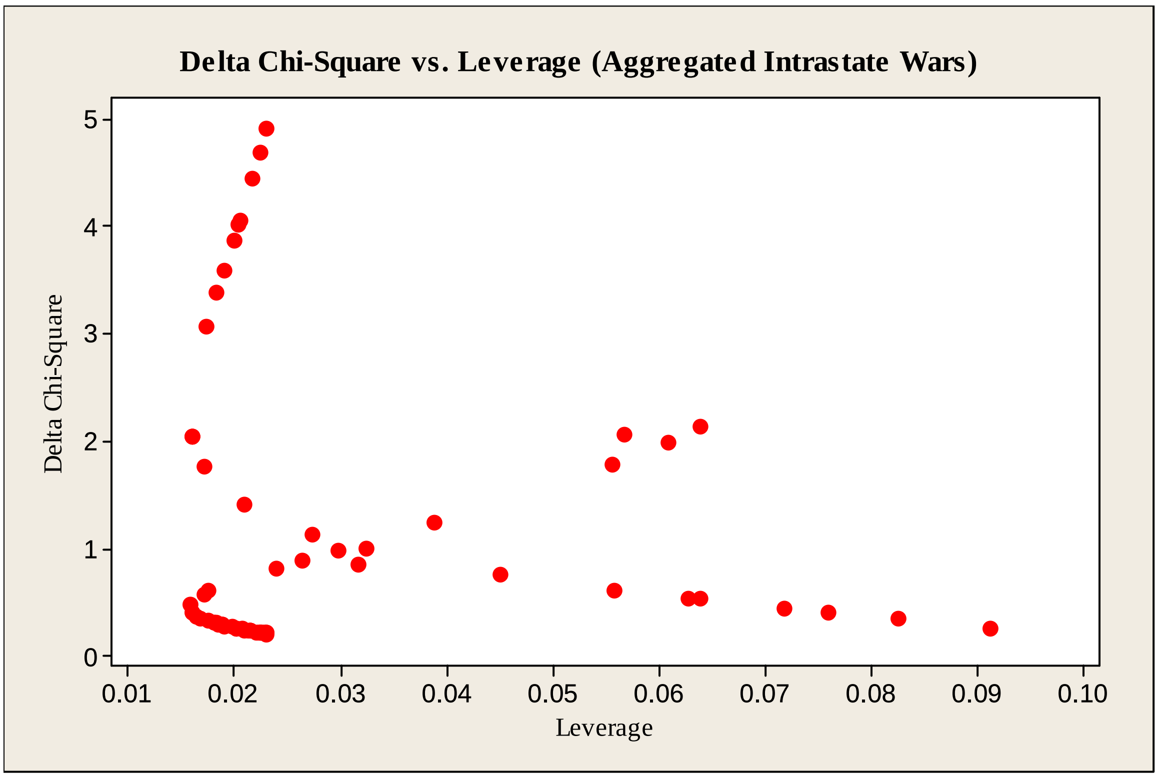

Figure 42: Pearson Chi-Square Change vs. Leverage Plot for Intrastate Wars

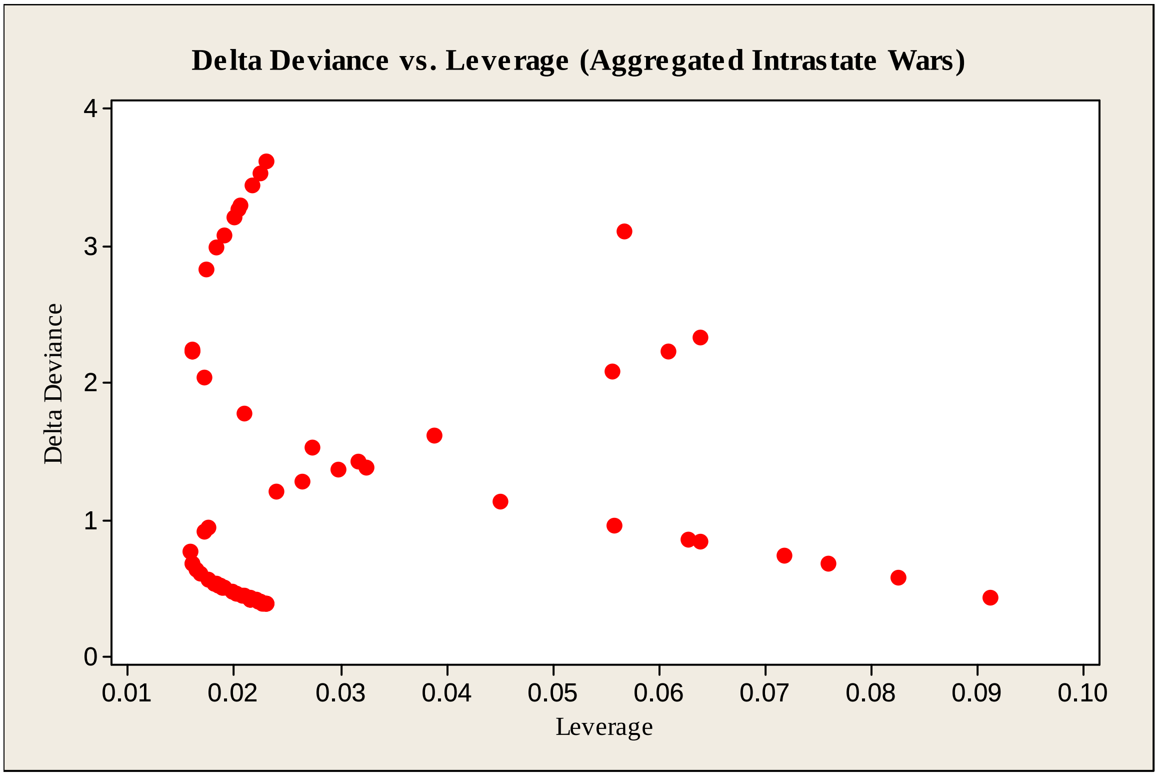

Figure 43: Deviance Change vs. Leverage Plot for Intrastate Wars

In contrast, the observation concerning the Somali rebels possessed the values $\Delta X_{59}^{2} = 2.06$, $\Delta D_{59} = 3.12$, and $h_{59} = 0.06$. Its estimated probability, however, lied on the range $0.3 < \hat{\pi}\left(x_{59}\right) < 0.7$. These values, with the exception of its estimated probability, were larger than those for the aforementioned observation, and its leverage fell within a cluster of 11 data points whose leverages were considered large in comparison to those of the remaining 62 observations. Therefore, this observation was not only an influence point, but it also exerted a greater amount of leverage on the estimation of the model coefficients than did the aforementioned observation.

Overall, the aggregated model for intrastate wars was considered to be a good predictor of the winner. It was not necessary to delete the observations identified from

the diagnostic plots and estimate a new model, because their respective values of $\Delta\hat{\beta}_j$,

ΔD_j, and ΔX_j^2 were not large enough to justify such an action. However, additional investigations into the aforementioned influential wars may be necessary to determine the nature of their effects on the model presented in this study.

Multinomial Logistic Regression Models on Outcome

Two multinomial models were estimated for predicting the outcome of an interstate war. An initial model for the 20th Century interstate wars data set contained the covariates for conflict duration and the proportion of total deaths borne by the participant, or *Duration_UNS_20* and *Dths/TDeaths_UNS_20*. The two covariates included in the initial 20th Century model resulted from the stepwise selection recommendation. Examination of their Wald statistics determined which, if not both, covariates was truly significant to the interstate wars model at the $\alpha = 0.05$ level. The model for the aggregated interstate wars contained only one covariate: *Deaths/TotDeaths_UNS*. The results for the 20th Century data are presented first. The Pearson chi-square and Deviance goodness-of-fit tests were computed for each of these multinomial models.

The ultimate objective of this investigation was to demonstrate the applicability of multinomial logistic regression to war termination studies. The summary figures from the MINITAB outputs were considered sufficient to accomplish this goal. Each figure contains the coefficient value, standard error of the coefficient, Wald statistic, p-value of the Wald statistic, odds ratio, and 95% confidence limits on the odds ratio for each of the covariates in each of the logits in the multinomial model. The frequency of each

outcome can be found at the top of each figure. The log-likelihood, likelihood ratio statistic, p-value of the likelihood ratio statistic, Pearson chi-square statistic, p-value of the Pearson chi-square statistic, Deviance statistic, and p-value of the Deviance statistic for the multinomial model are given at the bottom of each figure.

Each logit was referenced to the first outcome, or *Victory by Military Imposition*. As such, each odds ratio was a comparison of the outcome in question to the reference outcome. The odds ratio quantified how much more or less likely the outcome in question was to occur than the reference outcome, given a unit increase in the covariate values. The odds ratios were important to detecting patterns within the COWP data.

20th Century Interstate Wars

The initial model for the 20th Century data was bivariate. This model was estimated in response to the results from the stepwise selection procedure. The goodness-of-fit statistics and the individual Wald statistics were examined to determine if the initial model was sufficient to warrant further analysis. The initial model results are given in Figure 44.

The p-values for the two goodness-of-fit statistics were very high, which suggested the initial model to be adequately estimated. This was expected, in light of the results from the stepwise procedure in Chapter II. The p-value for the likelihood ratio statistic was smaller than 0.001, which rejected the null hypothesis in equation (2.29) and suggested that at least one $\hat{\beta}_j$ was nonzero.

The p-values for the Wald statistics, however, indicated that only one of the covariates was significant to the model at the 0.05 significance level. Each of the p-

values for the Wald statistics concerning *Dths/TDeaths_UNS_20* was smaller than 0.001. This implied that the proportion of total combat deaths sustained by the participant should be the only covariate, among the available COWP data, included in a multinomial model for 20th Century interstate wars. In contrast, each of the Wald statistic p-values for the duration of the conflict was larger than α. Thus, a reduced model containing only *Dths/TDeaths_UNS_20* was estimated. The implication from the statistics in Figure 44 was that the duration of an interstate war was not important to the outcome of a 20th Century interstate conflict. The length of the war may actually be important, but the COWP data did not reveal such a trend. Therefore, it should be stated that additional studies into interstate wars using other data sources may be necessary to identify other relevant variables which were not available in the COWP data.

Figure 45 shows the results from fitting the reduced multinomial model. Not only did both goodness-of-fit statistics show the model to be adequate, but also the Wald statistic p-value for *Dths/TDeaths_UNS_20* was smaller than 0.001, which implied that the single covariate maintained its significance to the model.

The odds ratios were interpreted individually. A one-unit change in *Dths/TDeaths_UNS_20* was defined for the purpose of interpreting the odds ratios. The standard deviation for the proportion of total deaths borne by the participant was computed to be 0.26, so each odds ratio was interpreted for an approximate 26% increase in *Dths/TDeaths_UNS_20*.

Nominal Logistic Regression: Outcome(PR2) versus Dths/TDeaths, Duration_UNS

Response Information

Variable	Value	Count	
Outcome(PR2)_20	1	62	(Reference Event)
	5	23	
	4	16	
	3	29	
	2	37	
	Total	167	

Logistic Regression Table

Predictor	Coef	SE Coef	Z	P	Odds Ratio	95%CI Lower	Upper
Logit 1: (5/1)							
Constant	-0.72514	0.286304	-2.53	0.011			
Dths/TDeaths_UNS_20	1.25622	0.305132	4.12	0	3.51	1.93	6.39
Duration_UNS_20	-0.24161	0.270117	-0.89	0.371	0.79	0.46	1.33
Logit 2: (4/1)							
Constant	-1.20834	0.351733	-3.44	0.001			
Dths/TDeaths_UNS_20	1.3249	0.334455	3.96	0	3.76	1.95	7.25
Duration_UNS_20	-0.598418	0.42479	-1.41	0.159	0.55	0.24	1.26
Logit 3: (3/1)							
Constant	-0.425834	0.259768	-1.64	0.101			
Dths/TDeaths_UNS_20	0.984011	0.292405	3.37	0.001	2.68	1.51	4.75
Duration_UNS_20	-0.118238	0.221858	-0.53	0.594	0.89	0.58	1.37
Logit 4: (2/1)							
Constant	-0.212314	0.247951	-0.86	0.392			
Dths/TDeaths_UNS_20	1.12896	0.278471	4.05	0	3.09	1.79	5.34
Duration_UNS_20	0.0069449	0.196298	0.04	0.972	1.01	0.69	1.48

Log-Likelihood = -231.512
Test that all slopes are zero: G = 39.155, DF = 8, P-Value = 0.000

Goodness-of-Fit Tests

Method	Chi-Square	DF	P
Pearson	624.98	636	0.615
Deviance	460.251	636	1

Figure 44: Results for Initial 20th Century Interstate Wars Model

Nominal Logistic Regression: Outcome(PR2)_20 versus Dths/TDeaths_UNS_20

Response Information

Variable	Value	Count	
Outcome(PR2)_20	1	62	(Reference Event)
	5	23	
	4	16	
	3	29	
	2	37	
	Total	167	

Logistic Regression Table

					Odds	95%	CI
Predictor	Coef	SE Coef	Z	P	Ratio	Lower	Upper
Logit 1: (5/1)							
Constant	-0.729073	0.285224	-2.56	0.011			
Dths/TDeaths_UNS_20	1.30012	0.303297	4.29	0	3.67	2.03	6.65
Logit 2: (4/1)							
Constant	-1.1367	0.328547	-3.46	0.001			
Dths/TDeaths_UNS_20	1.40105	0.329295	4.25	0	4.06	2.13	7.74
Logit 3: (3/1)							
Constant	-0.433706	0.259406	-1.67	0.095			
Dths/TDeaths_UNS_20	1.01319	0.291191	3.48	0.001	2.75	1.56	4.87
Logit 4: (2/1)							
Constant	-0.207563	0.246254	-0.84	0.399			
Dths/TDeaths_UNS_20	1.14257	0.277783	4.11	0	3.13	1.82	5.4

Log-Likelihood = -233.321
Test that all slopes are zero: G = 35.536, DF = 4, P-Value = 0.000

Goodness-of-Fit Tests

Method	Chi-Square	DF	P
Pearson	543.695	556	0.637
Deviance	407.684	556	1

Figure 45: Summary of Results for 20th Century Interstate Wars

In Logit 1, the outcome *Defeat by Negotiated Settlement* was compared to the reference outcome *Victory by Military Imposition*. Its odds ratio was 3.67, which was expressed using equation (2.38).

$$\hat{O}_{R5} = \frac{P(Y=5 \mid x=i) \big/ P(Y=1 \mid x=i)}{P(Y=5 \mid x=i+0.26) \big/ P(Y=1 \mid x=i+0.26)} = 3.67 \tag{4.9}$$

In other words, a participant in an interstate war is about three and a half times more likely to lose the war through a negotiated settlement than he is to win through military imposition, assuming that he bears more than one quarter of the total casualties.

In Logit 2, the outcome *Victory by Negotiated Settlement* was compared to the reference outcome. With an odds ratio of 4.06, equation (2.38) became

$$\hat{O}_{R4} = \frac{P(Y=4 \mid x=i) \big/ P(Y=1 \mid x=i)}{P(Y=4 \mid x=i+0.26) \big/ P(Y=1 \mid x=i+0.26)} = 4.06\,. \tag{4.10}$$

That is, an interstate war actor is about four times more likely to win the war through a negotiated settlement than through military imposition, assuming that he bears more than one quarter of the total casualties.

In Logit 3, the outcome *Stalemate* was compared to the reference outcome. Its odds ratio was 2.75, so equation (2.38) became

$$\hat{O}_{R3} = \frac{P(Y=3 \mid x=i) \big/ P(Y=1 \mid x=i)}{P(Y=3 \mid x=i+0.26) \big/ P(Y=1 \mid x=i+0.26)} = 2.75\,. \tag{4.11}$$

Therefore, an interstate war participant is 2.75 times more likely to accept the war as a stalemate than he is to win it by military imposition, assuming that he bears more than one quarter of the total casualties.

In the fourth and final logit, the outcome Capitulation was compared to the reference outcome. It possessed a 3.13 odds ratio, which was substituted into equation (2.38).

$$\hat{O}_{R4} = \frac{P\left(Y=2 \mid x=i\right) \Big/ P\left(Y=1 \mid x=i\right)}{P\left(Y=2 \mid x=i+0.26\right) \Big/ P\left(Y=1 \mid x=i+0.26\right)} = 3.13 \qquad (4.12)$$

Thus, a participant in an interstate war is over three times more likely to capitulate to the demands of his enemy than he is to win the war through military imposition, assuming that he bears more than one quarter of the total casualties.

It was interesting to notice that $\hat{O}_{R4} = 4.06$ was the largest of the odds ratios. It can be said that a nation involved in an interstate war is most likely to be on the side that wins through a negotiated settlement rather than win by military imposition, provided that the nation in question bears no more than one quarter of the total combat deaths. In other words, once a belligerent in an interstate war has taken about 26% of the total casualties, he should begin the process of negotiations to end the war on terms more favorable to him than to his enemy. This appeared to be the trend when 20th Century interstate wars were considered alone.

Aggregated Interstate Wars Model.

The stepwise selection procedure in Chapter II suggested that an aggregated interstate wars multinomial model be univariate. This recommendation left no room for a reduced model, so the univariate model was estimated with *Deaths/TotDeaths_UNS* as the single covariate. The Pearson chi-square and Deviance goodness-of-fit tests were

examined to assess overall model adequacy, and the p-value for the Wald statistic in each of the four logits was examined to determine the significance of *Deaths/TotDeaths_UNS*. Each of the four odds ratios was also interpreted to identify the most likely outcome for a nation involved in an interstate war, given that the nation has accepted a certain percentage of the total battle deaths. Figure 46 shows the MINTAB output for this multinomial model.

The p-values for both goodness-of-fit tests were much larger than $\alpha = 0.05$, which implied that the model was adequate. Each of the Wald statistic p-values was much smaller than $\alpha = 0.05$, which confirmed additionally that the covariate *Deaths/TotDeaths_UNS* was highly significant to the multinomial model. In fact, its Wald statistic p-value in all but one of the logits was very close to zero.

A one-unit change in *Deaths/TotDeaths_UNS* had to be defined for the purpose of interpreting the odds ratios. Because unit normal scaling was the data scaling technique used, the sample standard deviation for all $n = 225$ observations of *Deaths/TotDeaths_UNS* was defined as a single-step change in the value of the covariate. The sample standard deviation for the proportion of total deaths borne by the participant was computed to be 0.258, so each odds ratio was again interpreted for an approximate 26% increase in *Deaths/TotDeaths_UNS*. As with the 20^{th} Century model, the reference outcome for the aggregated model was also *Victory by Military Imposition*, or category 1.

Nominal Logistic Regression: Outcome(PR2) versus Deaths/TotDeaths_UNS

Response Information

Variable	Value	Count	
Outcome(PR2)	1	87	(Reference Event)
	5	28	
	4	26	
	3	31	
	2	53	
	Total	225	

Logistic Regression Table

Predictor	Coef	SE Coef	Z	P	Odds Ratio	95% CI Lower	Upper
Logit 1: (5/1)							
Constant	-1.1022	0.238411	-4.62	0			
Deaths/TotDeaths_UNS	0.975947	0.236328	4.13	0	2.65	1.67	4.22
Logit 2: (4/1)							
Constant	-1.13879	0.239475	-4.76	0			
Deaths/TotDeaths_UNS	0.880387	0.241627	3.64	0	2.41	1.5	3.87
Logit 3: (3/1)							
Constant	-0.915293	0.218258	-4.19	0			
Deaths/TotDeaths_UNS	0.666398	0.23343	2.85	0.004	1.95	1.23	3.08
Logit 4: (2/1)							
Constant	-0.393903	0.18603	-2.12	0.034			
Deaths/TotDeaths_UNS	0.760406	0.202088	3.76	0	2.14	1.44	3.18

Log-Likelihood = -321.021

Test that all slopes are zero: G = 28.353, DF = 4, P-Value = 0.000

Goodness-of-Fit Tests

Method	Chi-Square	DF	P
Pearson	688.882	696	0.569
Deviance	524.608	696	1

Figure 46: Results for Aggregated Interstate Wars Model

In Logit 1, the outcome *Defeat by Negotiated Settlement* was compared to the reference outcome *Victory by Military Imposition*. Its odds ratio was 2.65, which was expressed using equation (2.38).

$$\hat{O}_{R5} = \frac{P(Y=5 \mid x=i) \big/ P(Y=1 \mid x=i)}{P(Y=5 \mid x=i+0.26) \big/ P(Y=1 \mid x=i+0.26)} = 2.65 \tag{4.13}$$

In other words, a participant in an interstate war is over two and a half times more likely to lose the war through a negotiated settlement than he is to win through military imposition, assuming that he bears more than one quarter of the total casualties.

In Logit 2, the outcome *Victory by Negotiated Settlement* was compared to the reference outcome. With an odds ratio of 2.41, equation (2.38) became

$$\hat{O}_{R4} = \frac{P(Y=4 \mid x=i) \big/ P(Y=1 \mid x=i)}{P(Y=4 \mid x=i+0.26) \big/ P(Y=1 \mid x=i+0.26)} = 2.41 \qquad (4.14).$$

That is, an interstate war actor is nearly two and a half times more likely to win the war through a negotiated settlement than through military imposition, assuming that he bears more than one quarter of the total casualties.

In Logit 3, the outcome *Stalemate* was compared to the reference outcome. Its odds ratio was 1.95, so equation (2.38) became

$$\hat{O}_{R3} = \frac{P(Y=3 \mid x=i) \big/ P(Y=1 \mid x=i)}{P(Y=3 \mid x=i+0.26) \big/ P(Y=1 \mid x=i+0.26)} = 1.95\,. \tag{4.15}$$

Therefore, an interstate war participant is nearly two times more likely to accept the war as a stalemate than he is to win it by military imposition, assuming that he bears more than one quarter of the total casualties.

In the fourth and final logit, the outcome *Capitulation* was compared to the reference outcome. It possessed a 2.14 odds ratio, which was substituted into equation (2.38).

$$\hat{O}_{R2} = \frac{P(Y=2 \mid x=i) \big/ P(Y=1 \mid x=i)}{P(Y=2 \mid x=i+0.26) \big/ P(Y=1 \mid x=i+0.26)} = 2.14 \qquad (4.16)$$

Thus, a participant in an interstate war is over two times more likely to capitulate to the demands of his enemy than he is to win the war through military imposition, assuming that he bears more than one quarter of the total casualties.

The largest odds ratio of 2.65 implied that a nation would most likely be defeated through a negotiated settlement, assuming that the nation in question bore more than one quarter of the total casualties. A stalemate turned out to be the least likely outcome for the same conditions. The switch from victory to defeat by negotiated settlement between the 20th Century and aggregated analyses likely resulted from the effects that the 19th Century data had on the odds ratio calculations in the aggregated model. Approximately 88% of the 19th Century interstate wars identified in the COWP data ended by force of arms. This proportion dropped to 69% when the interstate wars from both centuries were considered together. One might conclude that a far greater prominence was placed on military force in the 19th Century than in the 20th Century.

A general trend of ending interstate wars by a negotiated settlement presented itself through the analyses of all interstate wars in the COWP data and the 20th Century interstate wars alone. This result supports a similar assertion made by Walker in his Naval War College study (Walker, 1996:1). It was also interesting to note that the casualty proportions necessary for prompting both outcomes were virtually equal. Thus,

a nation involved in an interstate war should move quickly for a favorable negotiated settlement once it sustains more than one quarter of all combat deaths.

Summary

The results in this chapter demonstrated that logistic regression techniques can be successfully applied to war termination problems. Stepwise selection fulfilled its usual purpose as a robust technique for identifying the covariates necessary to build an adequate logistic regression model on the response. For the 19th Century, 20th Century, and aggregated data on extra-systemic, intrastate, and interstate wars, the stepwise regression results were examined for accuracy. No logistic regression models for the 19th Century COWP data on any of the three types of wars were estimated because of the results from stepwise regression. Consequently, two models were fit for each war type: one for the 20th Century COWP data and one for the aggregated COWP data.

The final models estimated from extra-systemic war data were found to be good predictors of the winner. The models were parsimonious, and the winner was dependent only on the length of the war. Interpretation of their odds ratios revealed that the non-state belligerent was most likely to win a long extra-state war than the state actor. The United States has been engaged in the current war in Iraq for nearly four years, which is longer than the 1426-day duration change identified by the models. The Franco-Tonkin War of 1873, the Italo-Libyan War of 1920 and the Indonesian War of 1945 were found to be influential to the estimation of model parameters. Future statistical studies of these wars using a source with more complete and comprehensive data may reveal the reasons for their influences on the models in this study.

The two models estimated from the COWP data on intrastate wars were also good predictors of the winner. Again, the duration of the conflict was found to be the only available covariate significant to predicting the winner. The odd ratios for these models showed that the insurgent faction was even more likely to win an intrastate war than they were an extra-systemic war. However, the war duration requirement was longer than that for the extra-state models, about four and a half years. The influential secession movements and rebellions identified from the diagnostic plots of both models could be subjects of future investigations for further insights into their influence on the results of this study.

A general trend of ending interstate wars by a negotiated settlement presented itself through the results of both the 20th Century and aggregated models. As was the case with the models on extra-systemic and intrastate wars, the final multinomial models on interstate wars were also univariate. The single covariate significant to predicting the outcome of an interstate war, however, was not the length of the war but the percentage of total casualties sustained by a participating nation. The odds ratios from both models implied that an interstate war participant should seek a favorable negotiated peace once he has incurred more than 25% of the total battle deaths.

V. Discussion

Assessment of Current Findings

No models were fit using any of the 19th Century data. As a result, little can be said statistically regarding shifts in war termination trends between centuries. On the other hand, the degree to which the parameters, significance tests, and odds ratios differed between the 20th Century and aggregated models did demonstrate the amount of influence that 19th Century wars exerted on overall war termination trends.

It was interesting to see that the length of the conflict was most relevant for both intrastate and extra-state wars. The odds ratios between the 20th Century and aggregated extra-state wars model revealed a pattern favoring the insurgency faction over time. The non-state actor was over three times more likely to win when the 20th Century data were considered separately. This likelihood decreased for the aggregated model, and the insurgency became less than two times as likely to win. Thus, when duration is considered alone, an insurgency is more likely to win a prolonged war than the state which it is fighting.

The proportion of the total number of combat deaths borne by a nation involved in an interstate war was the most relevant variable for both multinomial models concerning interstate wars. Each outcome was referenced to the most frequent outcome of victory through force of arms. It was discovered that the odds ratios for the remaining outcomes were larger when the 20th Century data were considered alone than when the entire data set was analyzed. The implications for each case, however, were different. Given that a participating nation took about 26% of the total casualties, that nation was more likely to

win a 20th Century interstate war through a negotiated settlement than through military imposition. Pillar reached a similar conclusion by stating that explicit agreements are the most common form of terminating interstate wars (Pillar, 1983:16-17). His assertion, however, is broad in the sense that he grouped wars ending in imposed settlements and wars ending by negotiated settlements together, whereas this research analyzed these two outcomes separately.

The odds ratios for the aggregated interstate wars model were not as different from each other as those for the 20th Century model. Negotiated settlements still proved prevalent, as defeat and victory by negotiated settlement possessed the largest odds ratios of 2.65 and 2.41, respectively. The proportion of total casualties necessary for the likelihood of these outcomes was only slightly less than that for the 20th Century model, at about 25%. The pattern identified here was that nations involved in modern interstate wars could accept larger shares of the total casualties and still emerge victorious through negotiations than could those nations from 19th Century interstate wars.

Opportunities for Future Research

Advanced statistical techniques may be applied to the diagnostic results from this research. Specifically, the extra-state and intrastate wars identified as influential to model estimation could be tagged for more in-depth studies. Case-study approaches for these wars may help address the question of why these wars proved so influential in this research. This may be especially important when studying wars that have historically received scant attention.

The Italo-Libyan War of 1920, the Indonesian War of 1945, the Western Saharan War of 1975, and the Franco-Tonkin War of 1873 were identified in this research as influential to estimating the extra-systemic wars models. These wars were geographically focused in Africa and Southeast Asia, which may prove significant in discriminant studies on extra-state wars. Rather than emphasizing the importance of geography, one might discriminate between the combatants in these wars. The combat records of these belligerents may be of interest. Perhaps a multiple discriminant analysis (MDA) could be performed on both combatants and geography of these wars.

The Cambodia-Khmer Rouge War of 1970, the Pinochet Rebellion in Chile in 1973, the Somali Secession from Ethiopia in 1976, the Communist Rebellion in El Salvador in 1979, the Renamo Rebellion in Mozambique in 1979, and the Russo-Circasian War of 1829 were influential to estimating the intrastate wars model. Case-studies on these wars may provide additional insights into the reasons for their influences in this research. Opportunities for discriminant analyses also exist for these wars. One might investigate the factors that separate civil wars from secession wars.

With the United States engaged in the Global War on Terror (GWOT), which can be considered an extra-systemic war or series of extra-systemic wars, future studies on conventional interstate wars might not prove as significant to contemporary military operations as would studies on intrastate and extra-state wars. However, additional applications of logistic regression techniques exist for interstate wars. Additional relevant variables would need to be identified in order to expand upon the univariate main effects models presented in this thesis. Instead of a single multinomial logistic regression model, one might pair the possible outcomes of interstate wars and construct

binary logistic regression models for each pair. Using this approach, one might accurately identify influential interstate wars that warrant further statistical studies.

Appendix A.

Table 9: Variables and Definitions for COWP Interstate Wars Set

WarNo	War number
StateNum	COW country code of participant
StateAbb	Abbreviated name of participant
YrBeg1	First beginning year of participant's involvement
MonBeg1	First beginning month of participant's involvement
DayBeg1	First beginning day of participant's involvement
YrEnd1	First ending year of participant's involvement
MonEnd1	First ending month of participant's involvement
DayEnd1	First ending day of participant's involvement
YrBeg2	Second beginning year of participant's involvement (-999 = NA)
MonBeg2	Second beginning month of participant's involvement (-999 = NA)
DayBeg2	Second beginning day of participant's involvement (-999 = NA)
YrEnd2	Second ending year of participant's involvement (-999 = NA)
MonEnd2	Second ending month of participant's involvement (-999 = NA)
DayEnd2	Second ending day of participant's involvement (-999 = NA)
Duration	Length of war participation in days
Deaths	Number of battle related deaths sustained by participant's armed forces in war (-999 = missing)
Outcome	War outcome for participant (1 = on winning side, 2 = on losing side, 3 = on side A of a tie, 4 = on side B of a tie, 5 = on side A of an ongoing war, 6 = on side B of an ongoing war)
Initiate	Did state initiate war? (0 = no, 1 = yes)
SysStat	System membership status of state (1 = neither central sub-system member nor major power, 2 = central sub-system member only [only relevant 1816 through 1919], 3 = central sub-system member & a major power [only relevant 1816 through 1919], 4 = major power only)
PrWarPop	Pre-war population in thousands (number from year war begun, -999 = missing)
PrWarArm	Pre-war armed forces in thousands (number from year war begun, -999 = missing)
WestHem	Did state participant engage in fighting in war in Western Hemisphere? (0 = no, 1 = yes)
Europe	Did state participant engage in fighting in war in Europe? (0 = no, 1 = yes)
Africa	Did state participant engage in fighting in war in Africa? (0 = no, 1 = yes)
MidEast	Did state participant engage in fighting in war in Middle East? (0 = no, 1 = yes)
Asia	Did state participant engage in fighting in war in Asia? (0 = no, 1 = yes)
Oceania	Did state participant engage in fighting in war in Oceania? (0 = no, 1 = yes)
Version	Version number of data set

Table 10: Variables and Definitions for COWP Extra-Systemic Wars Set

WarNo	War number
StateNum	COW country code of participant
StateAbb	Abbreviated name of participant
YrBeg1	First beginning year of participant's involvement
MonBeg1	First beginning month of participant's involvement (-999 = missing)
DayBeg1	First beginning day of participant's involvement (-999 = missing)
YrEnd1	First ending year of participant's involvement
MonEnd1	First ending month of participant's involvement (-999 = missing)
DayEnd1	First ending day of participant's involvement (-999 = missing)
YrBeg2	Second beginning year of participant's involvement (-999 = NA)
MonBeg2	Second beginning month of participant's involvement (-999 = NA or missing)
DayBeg2	Second beginning day of participant's involvement (-999 = NA or missing)
YrEnd2	Second ending year of participant's involvement (-999 = NA)
MonEnd2	Second ending month of participant's involvement (-999 = NA or missing)
DayEnd2	Second ending day of participant's involvement (-999 = NA or missing)
MinDur	Minimum length of war participation in days*
MaxDur	Maximum length of war participation in days*
Deaths	Number of battle related deaths sustained by participant's armed forces in war (-999 = missing)
IntSide	On which side did participant intervene? (0 = NA/state is primary actor in war, 1 = on side of state; 2 = on side of colony/non-state, 3 = on neither side)
Initiate	Did state initiate war? (0 = no, 1 = yes)
SysStat	System membership status of state (1 = neither central sub-system member nor major power, 2 = central sub-system member only [only relevant 1816 through 1919], 3 = central sub-system member & a major power [only relevant 1816 through 1919], 4 = major power only)
PrWarPop	Pre-war population in thousands (number from year war begun, -999 = missing)
PrWarArm	Pre-war armed forces in thousands (number from year war begun, -999 = missing)
WestHem	Did state participant engage in fighting in war in Western Hemisphere? (0 = no, 1 = yes)
Europe	Did state participant engage in fighting in war in Europe? (0 = no, 1 = yes)
Africa	Did state participant engage in fighting in war in Africa? (0 = no, 1 = yes)
MidEast	Did state participant engage in fighting in war in Middle East? (0 = no, 1 = yes)
Asia	Did state participant engage in fighting in war in Asia? (0 = no, 1 = yes)
Oceania	Did state participant engage in fighting in war in Oceania? (0 = no, 1 = yes)
Version	Version number of data set

Table 11: Variables and Definitions for COWP Intrastate Wars Set

WarNo	War number
StateNum	COW country code of participant
StateAbb	Abbreviated name of participant
YrBeg1	First beginning year of participant's involvement
MonBeg1	First beginning month of participant's involvement (-999 = missing)
DayBeg1	First beginning day of participant's involvement (-999 = missing)
YrEnd1	First ending year of participant's involvement
MonEnd1	First ending month of participant's involvement (-999 = missing)
DayEnd1	First ending day of participant's involvement (-999 = missing)
YrBeg2	Second beginning year of participant's involvement (-999 = NA)
MonBeg2	Second beginning month of participant's involvement (-999 = NA or missing)
DayBeg2	Second beginning day of participant's involvement (-999 = NA or missing)
YrEnd2	Second ending year of participant's involvement (-999 = NA)
MonEnd2	Second ending month of participant's involvement (-999 = NA or missing)
DayEnd2	Second ending day of participant's involvement (-999 = NA or missing)
MinDur	Minimum length of war participation in days*
MaxDur	Maximum length of war participation in days*
Deaths	Number of battle related deaths sustained by participant's armed forces in war (-999= missing)
IntSide	On which side did participant intervene? (0 = NA/state is undergoing intra-state war, 1 = on side of state; 2 = on side of opposition, 3 = on neither side)
SysStat	System membership status of state (1 = neither central sub-system member nor major power, 2 = central sub-system member only [only relevant 1816 through 1919], 3 = central sub-system member & a major power [only relevant 1816 through 1919], 4 = major power only)
PrWarPop	Pre-war population in thousands (number from year war begun, -999 = missing)
PrWarArm	Pre-war armed forces in thousands (number from year war begun, -999 = missing)
Version	Version number of data set

Appendix B.

Table 12: Variables and Definitions for COWP MID Data Set

Variable Number	Variable Name	Variable Description		
1	*DispNum*	Dispute Number		
2	*StDay*	Start day of dispute (-9 = missing)		
3	*StMon*	Start month of dispute (-9 = missing)		
4	*StYear*	Start year of dispute (-9 = missing)		
5	*EndDay*	End day of dispute (-9 = missing)		
6	*EndMon*	End month of dispute (-9 = missing)		
7	*EndYear*	End year of dispute (-9 = missing)		
8	*Outcome*	Outcome of dispute:		
		1	Victory for side A	
		2	Victory for side B	
		3	Yield by side A	
		4	Yield by side B	
		5	Stalemate	
		6	Compromise	
		7	Released	
		8	Unclear	
		9	Joins ongoing war	
		-9	Missing	
9	*Settle*	Settlement of dispute:		
		1	Negotiated	
		2	Imposed	
		3	None	
		4	Unclear	
		-9	Missing	
10	*Fatality*	Fatality level of dispute:		
		0	None	
		1	< 26	deaths
		2	26-100	deaths
		3	101-250	deaths
		4	251-500	deaths
		5	501-999	deaths
		6	> 999	deaths
		-9	Missing	
11	*FatalPre*	Precise Fatalities, if known (-9 = missing)		
12	*MaxDur*	Maximum duration of dispute		
13	*MinDur*	Minimum duration of dispute		

Table 13: Variables and Definitions for MID Set (cont.)

Variable Number	Variable Name	Variable Description	
14	*HiAct*	Highest action in dispute [bracketed numbers refer to corresponding hostility level]:	
		0	No militarized action [1]
		1	Threat to use force [2]
		2	Threat to blockade [2]
		3	Threat to occupy territory [2]
		4	Threat to declare war [2]
		5	Threat to use CBR weapons [2]
		6	Threat to join war
		7	Show of force [3]
		8	Alert [3]
		9	Nuclear alert [3]
		10	Mobilization [3]
		11	Fortify border [3]
		12	Border violation [3]
		13	Blockade [4]
		14	Occupation of territory [4]
		15	Seizure [4]
		16	Attack [4]
		17	Clash [4]
		18	Declaration of war [4]
		19	Use of CBR weapons [4]
		20	Begin interstate war [5]
		21	Join interstate war [5]
		-9	Missing [-9]
15	*HostLev*	Hostility level of dispute:	
		1	No militarized action
		2	Threat to use force
		3	Display of force
		4	Use of force
		5	War

Table 14: Variables and Definitions for MID Set (cont.)

Variable Number	Variable Name	Variable Description
16	*Recip*	Reciprocated dispute? (1 = yes, 0 = no)
17	*NumA*	Number of states on side A
18	*NumB*	Number of states on side B
19	*Link1*	Links to other disputes/wars #1 (contains dispute number [variable "DispNum"] of other dispute; links to war indicated by code "W" e.g. "167W" is link to war number 167)
20	*Link2*	Links to other disputes/wars #2
21	*Link3*	Links to other disputes/wars #3
22	*Ongo2001*	Ongoing after 2001? (0 = concluded before 12/31/2001, 1 = continuing as of 12/31/2001
23	*Version*	Version number of data set

Appendix C.

Table 15: Variables and Definitions for National Materiel Capabilities

StateAbb	3 letter country abbreviation
Ccode	COW Country code
Year	Year of Observation
IrSt	Iron and steel production (thousands of tons)
MilEx	Military expenditures (thousands of 2001 US dollars)
MilPer	Military Personnel (thousands)
Energy	Energy consumption (thousands of coal-ton equivalents)
Tpop	Total Population (thousands)
Upop	Urban Population (population living in cities with population greater than 100,000)
CINC	Composite Index of National Capability (CINC) score
Version	Version number of the data set

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14. ABSTRACT

This research uses an advanced statistical technique to expand upon the current understanding of war termination. Specifically, this thesis addressed questions concerning the most relevant factors toward predicting both the outcomes of interstate wars and the winners of intrastate and extra-systemic wars, within the limitations of the available data. Open-source war data from the Correlates of War Project was analyzed using both binary and multinomial logistic regression techniques. While the Correlates of War Project did not necessarily focus its data collection efforts on those variables historically associated with war termination, it did provide a sufficient number of variables with which to demonstrate the applicability of logistic regression techniques to war termination analyses. As a consequence, every significant logistic regression model contains a single relevant variable. For both intrastate and extra-systemic wars, the duration of the conflict was found to be most relevant to predicting the winner. In contrast, the proportion of total casualties borne by a nation in an interstate war was most relevant to predicting the manner in which an interstate war ends. Conclusions drawn from this research and suggestions for future statistical applications to war termination studies were also discussed.

15. SUBJECT TERMS

War Termination, Statistical Analysis, Regression, Logistic Regression Analysis, Maximum Likelihood Estimation, Interstate War, Extra-systemic War, Intrastate War

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